Real Estate Return Correlations: Real-World Limitations on Relationships Inferred from NCREIF Data

by

Richard A. Graff
Principal
Electrum Partners
400 North Michigan Avenue, Suite 415, Chicago, Illinois 60611
phone: 312-923-8144 / fax: 312-923-8023

and

Michael S. Young
Vice President and Director of Quantitative Research
The RREEF Funds
101 California Street, San Francisco, California 94111
phone: 415-781-3300 / fax: 415-781-2229 / e-mail: MYoung@RREEF.com

Winner of Prize for "Best Paper Presented by a Practicing Real Estate Professional" at the 1994 Annual Meeting of the American Real Estate Society

published in

Journal of Real Estate Finance and Economics
Vol. 13, No. 2, September 1996, pp. 121-142

Real Estate Return Correlations: Real-World Limitations on Relationships Inferred from NCREIF Data

by
Richard A. Graff and Michael S. Young

Abstract: Correlation estimates for returns between individual properties are subject to large inherent uncertainties due to limits on the amount of data that is likely to be available for the foreseeable future. After allowance for correlation sampling error, it is impossible to distinguish on an *ex ante* basis between the risk reduction capabilities of mean-variance portfolio selection models and naive diversification without regard for property type or geographical location. The naive portfolio diversification strategies of typical institutional real estate portfolio managers are rational responses to limitations on the informational content of statistical analyses of historical real estate data.

Key Words: Correlation, portfolio diversification, sample error, z-transform, NCREIF data base

In a recent article, Young & Graff [1995] began an empirically-based reexamination of Modern Portfolio Theory (MPT) methodology in the commercial real estate context by presenting historical evidence that heteroscedastic stable infinite-variance distributions are better risk models for real estate investment than normal distributions. The study concluded that most real estate risk is diversifiable, but that investment strategies aiming at significant reduction in the diversifiable portfolio risk component require diversification across far more properties than are represented in typical institutional portfolios. This implied similarly that institutional portfolios designed to track published real estate indices require diversification across far more properties than deemed necessary by previous real estate portfolio research.

By contrast, nearly all previous real estate portfolio research has been based on the assumption that real estate risk is stationary with independent normally distributed samples, including the studies that formed the basis for quantitative diversification strategies developed for institutional commercial real estate portfolios in the 1980s and 1990s. In addition, previous research has suffered uniformly from the shortcoming of not fully considering the effects of sampling error on quantitatively derived results.

Since the portfolio diversification conclusions of previous studies contrast strongly with the results of Young & Graff [1995], it is natural to ask whether the conclusions differ primarily because of a different model for real estate investment risk or due to incomplete error analysis in the earlier studies. The present article investigates this question by examining informational limits on the effectiveness of elementary portfolio diversification strategies derived from MPT under the (conventional) assumption that real estate risk is stationary with independent normally distributed samples.

We present evidence that multifactor portfolio return/risk models based on property type and/or geographical location are virtually useless in the information they provide about the composition of "efficiently diversified" portfolios. More precisely, we show that individual property returns from the NCREIF data base do not support the existence of so-called economic geographical regions based on the following two criteria:

1. Property return behavior is relatively homogeneous *within* regions and relatively heterogeneous *across* regions;
2. The regions contain enough individual properties to form target investment subuniverses for quantitative portfolio diversification and optimization.

These conclusions follow straightforwardly from an examination of sample errors for correlations among NCREIF time series returns.[1]

The correlation function is the quantitative tool most commonly used by researchers to determine how to decompose an investment universe into relatively independent asset subgroups. The methodology in this study is simply to apply hypothesis testing to determine whether or not observed sample correlations imply that the corresponding true product-moment correlations are numerically distinct. This issue is of considerable practical consequence: if the true product-moment correlations cannot be shown to be unequal, then MPT diversification is equivalent on an *ex ante* basis to naive diversification without regard for property type and/or geographical region. In other words, the *ex ante* effectiveness of diversification is solely a function of the number of properties in the portfolio.[2] This implies that the incremental agency costs associated with portfolio diversification across property type and/or geographical region are unnecessary, as is the high-tech baggage associated with quadratic optimization.

Real estate portfolio studies routinely make the assumption that true correlation values differ merely because the corresponding sample correlations differ—an assumption that would earn a poor grade for any college undergraduate who did likewise on a statistics course final. In those few cases in which error analysis for the correlation function is considered, researchers (a) overestimate the number of independent correlation samples that may be assembled from each data set, and (b) incorrectly treat the correlation function as if it were normally distributed in confidence interval determination and hypothesis testing.[3] Such procedures lead to erroneous assumptions about correlation estimator accuracies, and in turn to incorrect qualitative conclusions about relations between various intraclass and interclass correlations.

It follows from the statistical methodology in this study that real estate correlation coefficient estimators contain much greater sample noise than portfolio strategists typically assume. This

1 The comparison of diversification strategy effectiveness indirectly via examination of sample errors in correlation matrix estimators has a decided advantage over direct comparisons of diversified portfolio returns: namely, hypothesis tests based on correlation matrices can be conducted without the need for any ancillary assumptions concerning the market pricing of real estate investment risk, cf. the Miles & McCue [1984a] investigation of real estate diversification strategies. That study did not use correlations but required the assumption of a one-parameter market model for commercial real estate.

2 Among other things, this implies that all divisions of the United States into economic geographical regions for purposes of efficient diversification are unsupported by NCREIF data when sample error is taken into account. This includes the so-called "Eight Nations of America" currently touted by researchers at several institutional investment managers as a fix for the failed MPT strategies promoted during the 1980s.

3 Normally distributed real-valued random variables can assume all values between $-\infty$ and $+\infty$. Since the range of each correlation function is the closed interval [-1,+1], it is apparent that correlations cannot be normally distributed.

implies that sample correlations should be expected to display major fluctuations as sample periods and/or sets of properties are varied, a prediction that can be tested empirically.[4]

Seven subsets of NCREIF individual property returns from six different sample periods are analyzed here. In every one of these cases, it will be shown that the sample correlation matrix is statistically indistinguishable from a correlation matrix with identical entries. Furthermore, individual sample correlations will be seen to display large fluctuations as the sample periods and property sets are varied, consistent with the predictions of the correlation noise model for stationary returns.

Since the NCREIF data base is significantly larger than the domestic property data base of any institutional real estate manager or investor (and includes most properties of the larger managers), the limitations determined by our analysis apply to all quantitative real estate diversification strategies developed over the past twenty years. Thus, the diversification conclusions of Young & Graff [1995] are consistent with the implications of diversification analysis based on a normal independent identically distributed risk model when the effect of sample noise is included.

Despite its heavy reliance on statistics, this article is a practical critique of real world quantitative portfolio practices. Accordingly, only elementary quantitative portfolio strategies are considered here because these are the only quantitative strategies thus far to have made the transition from the world of academic theory to the marketplace for institutional portfolio services.[5]

The basic principle underlying this analysis is intuitive: correlation estimates based on short time series are subject to large sample errors that translate into large uncertainties in quantitative portfolio diversification strategies. If longer time series are available—e.g., returns for publicly-traded REITs—then the focus of diversification strategy error analysis shifts from the correlation matrix to the underlying joint returns distribution, and the sample noise in optimal portfolio selection can be estimated by statistical bootstrapping techniques.[6]

Some of the ideas in this article appear in less developed form in Young & Greig [1993].

---

[4] If correlation confidence intervals were significantly smaller than those derived in this study, then large fluctuations in sample correlation observations would imply that real estate returns are not stationary. This would imply in turn that correlation estimates would be of historical interest only and useless for the prediction of future portfolio performance. Cf. Hartzell, Shulman & Wurtzebach [1987], which suggests that changes in observed correlation samples are consequences of fundamental shifts in real estate market conditions rather than sample noise, implying that commercial real estate returns are nonstationary. Despite the inherent contradiction, those authors suggest that their correlation estimates can be used to estimate future real estate portfolio performance.

[5] For example, the assumed independence of returns from different sample periods excludes portfolio strategies based on autocorrelated return models such as the disaggregate smoothing model introduced in Geltner, Graff & Young [1994]. Since it is not presently known how to construct useful statistical estimators for the smoothing parameters in such models, this exclusion does not constitute a practical restriction.

[6] Portfolio diversification noise is not quite as large in the general bootstrapping case (i.e., diversification turns out to depend on more than simply the number of assets in the portfolio), but limits on *ex ante* effectiveness of MPT diversification turn out to be qualitatively similar to the results derived here. See Graff, Young & Schoenberg [1995a,b] for applications to portfolios of publicly traded REITs (i.e., passive real estate investment) and asset allocation among publicly traded asset classes, respectively; cf. Liang, Myer & Webb [1994] for an application to directly owned real estate and asset allocation. For a concise survey of bootstrapping methodologies, see Efron & Tibshirani [1986].

Annual versus Quarterly Sampling Periods

When pension funds and financial institutions began to make significant commitments to equity commercial real estate, they faced the problem of valuing their investments on a quarterly basis for reporting purposes. Since most institutions invest in real estate indirectly through institutional real estate managers and quarterly portfolio management fees are based on portfolio value (which increases with successful manager performance), there was also significant manager incentive for quarterly property revaluations.

Because only a small percentage of institutional portfolio assets changes hands each year and property is not fungible, the pragmatic decision was made to use appraisal valuations and returns to satisfy reporting requirements and determine investment management fees, under the assumption that appraisal valuations are reasonably efficient and unbiased estimators of true market value. With the additional assumption that appraisal errors are independent and hence diversifiable, it also seemed that portfolio valuation accuracy would have to exceed individual property valuation accuracy, and that portfolio valuation accuracy would only improve as real estate portfolios were diversified more efficiently.

In an unrelated development, in the early 1980s institutional portfolio strategists and consultants began to apply MPT portfolio selection models to real estate portfolio management. These quantitative models, developed originally for application to common stock portfolios, are based on mean-variance optimization, and apply only to portfolio selection from investment classes that satisfy certain statistical criteria: return distributions must be i.i.d. (independent and identically distributed across sampling periods), with reasonably accurate statistical measurements of the return, risk, and codependence characteristics of the distributions available for all assets in the investment universe. In addition, asset returns are usually required to be approximately normally distributed.[7]

Since real estate portfolio strategists and consultants already had quarterly appraisal-based returns available (from NCREIF or individual investment managers), they were strongly motivated to use quarterly returns rather than annual returns in MPT portfolio selection and management models. This quadrupled the sample size for individual asset return series, apparently reducing standard sampling errors for each parameter estimate by a factor of two.

However, as many authors have noted, quarterly appraisal-based return series display properties inconsistent with the above requirements for MPT portfolio selection model input: nonnormality, seasonality, and autocorrelation.

For example, Wheaton & Torto [1989, p.442] observed that quarterly NCREIF index returns from 1978-1988 display a seasonal pattern, with most capital appreciation occurring in just one or two quarters each year. They concluded that property appraisals were not spread evenly throughout the year. As a result, they chose to work with annual returns rather than quarterly returns, even though (as they noted) this reduced the number of data samples.

Miles & McCue [1984b] examined individual property returns for a large commingled real estate fund, some of which are included in the NCREIF data base (see Hartzell, Hekman &

[7] Levy & Markowitz [1979] extended the apparent applicability of MPT portfolio selection to asset classes with arbitrary i.i.d. return series with finite mean and variance, under the mildly restrictive assumption that individual investor utility is capable of local approximation by quadratic utility. However, the problem remains that economists have not discovered a practical methodology to determine whether investor utility functions meet the local approximation criterion. Thus, in practice, approximate normality of asset returns is still the operative criterion for MPT applicability.

Miles [1986, p.233]). While properties in this fund were revalued each quarter, each property was reappraised only once per year by an outside appraiser, the other three revaluations coming from in-house appraisers. Miles and McCue observed (p. 359) that most of the changes in appraised value came from the outside appraisals, with inside appraisals often indicating no change in valuation. They reported that quarterly property return series displayed bumps, and that this produced positive skewness and kurtosis in the sample return distributions.[8]

Giliberto [1990, p.261] and Gyourko & Keim [1992, p.459] noted seasonal behavior and serial correlation respectively in the quarterly NCREIF indices, which they attributed to a relatively high number of property appraisals in the fourth quarter.

Liu, Hartzell & Grissom [1992] observed skewness in quarterly returns from several institutional commingled real estate funds, and developed statistical evidence that skewness could be a systematic real estate risk factor. Myer & Webb [1994] observed both skewness and positive kurtosis in quarterly NCREIF index returns. The kurtosis disappeared when they examined annual returns, although evidence of skewness persisted.

The analyses in Geltner [1993, Appendix B] and Geltner, Graff & Young [1994, Appendix] imply that temporal aggregation-induced smoothing dampens correlations between quarterly real estate subindices. On the other hand, Graff [1995] showed that the nonstationarity of quarterly returns generates: (a) upward bias in sample correlations between returns from individual properties subject to annual reappraisal during the same quarter, and (b) downward bias in sample correlations between returns from individual properties subject to annual reappraisal during different quarters. Each of these studies implies that sample correlations based on quarterly returns are not consistent estimators for true correlation parameters.

In short, the real estate economics literature contains numerous demonstrations of the nonstationarity/nonnormality of quarterly real estate returns, as well as analyses implying that sample correlations based on historical quarterly returns cannot predict relations between future quarterly returns. Thus, forward-looking correlation estimates for privately owned real estate (e.g., MPT portfolio selection model input) should only be derived from annual appraisal-based returns.

Methodology

The product-moment correlation coefficient is a numerical value ρ in the unit interval [-1, +1] that measures the degree of linear dependence between two random variables. For the definition to make sense, the two random variables must have finite expected values and variances. While in general the correlation coefficient is not observable, multiple pairs of random samples from the two random variables can be used to generate a correlation coefficient function/estimator (random variable) as follows:

$$r = r(f, g) = \mathrm{cov}(f, g)/(s_f s_g) \qquad (1)$$

where $\mathrm{cov}(f, g)$ is the sample covariance between *f* and *g*, and s_f and s_g are the sample standard deviations of *f* and *g* respectively. Note that the correlation function/estimator depends on the number of pairs of sample values for *f* and *g*.

---

8 Miles & McCue [1984b, Note 17] also noted that annual data series lack the problems of quarterly data, but indicated a preference for the quarterly data due to the greater number of quarterly observations. Their quarterly data was thenceforth treated as if stationary and normally distributed, with analyses performed and conclusions drawn accordingly.

Each such correlation estimator is commonly referred to by researchers as “he correlation function.” If the two random variables are components of an i.i.d. bivariate distribution then, under fairly general conditions, each correlation function is a consistent estimator for ρ. Because the range of the correlation function is contained in the interval [-1,+1], the distribution of the correlation function is decidedly nonnormal.[9]

If the true correlation value is contained in the open interval (-1,+1), then the probability is 100% that any finite set of correlation samples is also contained in the open interval (-1,+1). Conversely, if the value of even a single correlation sample is in the open interval (-1, +1), then the true correlation value must also be contained in the open interval (-1,+1).

The research design for this study is to examine confidence intervals for correlation values, and to test whether matrices of correlation sample values are distinct statistically from matrices with identical entries. If not, then it will follow that mean-variance portfolio selection models based on sample correlation matrices cannot be distinguished *ex ante* from mean-variance portfolio selection models based on correlation matrices with identical entries. Since portfolio selection models based on correlation matrices with identical entries are the same as so-called naive diversification models (asset selection without concern for the correlation of returns between individual portfolio assets), it will follow immediately that the effectiveness of real estate MPT portfolio diversification models cannot be distinguished from the effectiveness of naive diversification.

If correlation functions were normally distributed, these tests would be straightforward applications of standard error estimates and chi-squared tests for normally distributed random variables. Although correlation functions are not themselves normally distributed, R.A. Fisher [1915] introduced a nonlinear transformation of correlation functions into random variables that are approximately normal. This transformation, known as the z-transformation, reduces the complexity of standard error estimates and chi-squared tests involving correlations to the corresponding estimates and calculations for normally distributed random variables. The rest of this section is devoted to a summary of the properties of the z-transformation that are needed for the applications in this study.

The z-transformation is a nonlinear transformation of the correlation function defined as follows:

$$r = \tanh(z) \tag{2}$$

where $\tanh()$ is the hyperbolic tangent function, so that:

$$z = \text{invtanh}(r) = (1/2)\ln\left[(1+r)/(1-r)\right] \tag{3}$$

The z-transform of the correlation function has a range of $(-\infty,+\infty)$. Under the additional assumption that f and g are normal and the components of a bivariate normal distribution, the correlation function has finite mean and variance, an approximately normal distribution that becomes more normal as the number of samples increases, and a mean value very nearly equal to the z-transform of the true correlation value ρ. Furthermore, the variance is nearly independent of the true value of the correlation, and is approximated by the following simple equation:

$$\text{var}(z) = 1/(N - 8/3) \tag{4}$$

---

[9] See Hotelling [1953] for correlation function distributions in the case of normally distributed random variables, and Gayen [1951] in the case of nonnormal random variables with finite moments up through order four.

where N is the number of independent pairs of sample values for f and g used to define the correlation estimator.[10] Finally, the z-transform is robust to deviations of the joint distribution of f and g from normality, which implies that Equation (4) can be applied to estimate the correlation variance in many situations in which the bivariate normality assumption may be only a rough approximation to the true joint distribution of the random variables.[11]

It follows from Equation (4) that it is not necessary to have more than one correlation sample in order to generate a correlation confidence interval, nor any additional information other than the number of pairs of return samples used to generate the correlation sample. The procedure for generating an efficient 95% confidence interval for the correlation ρ between f and g from N pairs of sample values for the two functions is straightforward. First, let r_0 be the correlation estimate given by Equation (1), let z_0 be the z-transformation of r_0, and let z be the z-transform of ρ. Because the z-transformation is very nearly normal, a 95% confidence interval for ζ conditioned on the estimate r_0 is given by $(z-1.96\sigma, z+1.96\sigma)$, where $\sigma = 1/\sqrt{N-8/3}$. Letting $z_l = z-1.96\sigma$ and $z_u = z+1.96\sigma$, the corresponding 95% confidence interval for ρ conditioned on the estimate r_0 becomes $(\tanh(z_l), \tanh(z_u))$.

Let us now assume that there are M independent sample correlation values $r_1,\ldots,r_M$ available, each of which has been obtained from N pairs of values for f and g. Then the appropriate procedure in computing a 95% confidence interval for ρ is, roughly, to compute the z-transformations z_j of r_j for $j=1,\ldots,M$, let $\bar{z}$ be the average of the z_j, and note that $\bar{z}$ is a sample value for the average of M identical independent random variables, the expected value of which is ζ. Thus, in this case, a 95% confidence interval for ζ is very nearly given by $(\bar{z}-1.96\sigma/\sqrt{M}, \bar{z}+1.96\sigma/\sqrt{M})$, with a 95% confidence interval for ρ obtained by taking the inverse z-transformation of the end points of this interval. However, we must caveat this calculation slightly. In some cases there may be a complication due to the fact that, while the z-transformation is very nearly normally distributed, the mean of the distribution is biased away from ζ by $\rho/(2N-2)$.

The bias in the z-transformation is usually ignored in the case first discussed (confidence intervals when only one sample correlation value is available), since the bias in the expected z-value is significantly smaller in magnitude than the radius of the confidence interval for ζ. In addition, determination of a precise value for the bias is complicated by the fact that it depends on the true value ρ for the correlation parameter, which is *a priori* unknown. However, if $M \geq 16(N-8/3)$, then $1.96\sigma/\sqrt{M} < 1/(2(N-8/3))$, which implies that the magnitude of the bias in the ζ estimate could be as large as the radius of the confidence interval for ζ. Furthermore,

---

10 Var(z) is assumed in most references to be 1/(N-3), as originally suggested by Fisher [1921]. Hotelling [1953] showed that the variance estimate in Equation (4) is an improvement over 1/(N-3) for very small values of N, and observed that for larger N the difference between the two estimates is insignificant. Gayen [1951, Sec. 8] showed that, for N≥11, the variance estimate 1/(N-3) and the estimate inv tanh(r) for the z-transform mean are close to optimal estimates and the z-transform is for all practical purposes normally distributed. Lin & Mudholkar [1980] showed that the z-transform is very close to normally distributed for small values of ρ under the milder restriction N≥5.

11 Gayen [1951] showed that z-transform remains approximately normal in the case of correlations between nonnormally distributed random variables with finite moments, even if the variables are highly skewed. However, the simple estimators for the z-transform mean and variance that are valid for the case of normally distributed variables require larger sample sizes in order to remain valid for the nonnormal case. Gayen showed that a sample size of N≥21 is almost always sufficient, under the additional assumption that |ρ|≤0.8 (where ρ is the true correlation value). Pearson [1929, pp. 356-60] and Chesire, Oldis & Pearson [1932] showed that N≥10 often suffices (again, under the additional assumption that |ρ|≤0.8).

the uncertainty in the true value of ρ is reduced, due to the reduction in the width of the confidence interval for ζ by the factor $\sqrt{M}$. This implies in turn that the true value of the bias can be estimated with more certainty than in the case of one sample correlation value. Thus, in this case, an improved ζ estimate is given by $\bar{\bar{z}} = \bar{z} - \tanh(\bar{z})/(2N-2)$.[12] Letting $z_l = \bar{\bar{z}} - 1.96\sigma/\sqrt{M}$ and $z_u = \bar{\bar{z}} + 1.96\sigma/\sqrt{M}$, the corresponding best estimate r_0 for ρ is given by:

$$r_0 = \tanh\left(\bar{\bar{z}}\right) \tag{5}$$

with the corresponding 95% confidence interval given by:

$$(r_l, r_u) = (\tanh(z_l), \tanh(z_u)) \tag{6}$$

It is important to emphasize that, in order for *M* independent correlation samples to be computed from *N* pairs of samples for random variables *f* and *g*, it is necessary to have *M* independent sets of *N* samples for *f* and also *M* independent sets of *N* samples for *g*.

For example, if the object is to examine correlations of annual returns between office buildings and industrial buildings during the 1980s, and if we have returns for nine office buildings but only four industrial buildings, then the largest value that *M* can have is four.[13] While thirty-six separate sample correlations can be computed, these thirty-six samples are not independent. Thus, it is perfectly acceptable to use all of the information available and estimate $\bar{z}$ or $\bar{\bar{z}}$ by averaging the z-transformations of all thirty-six sample values; however, due to the fact that no more than four correlation samples can be independent, the radius of the confidence interval of the ζ estimate can only be reduced by a factor of $2 = \sqrt{4}$, not by a factor of $6 = \sqrt{36}$. By the same rationale, the largest value that *M* can have for (intraclass) office-office correlations in this example is four, and for industrial-industrial correlations is two.[14]

---

12 See Fisher [1921].

13 This may still overstate the number of independent samples available. The true number of independent samples in this example may be less than four—in fact, it may be only one. If either the true office-office correlation or the true industrial-industrial correlation is nonzero, then the vector-valued pair of returns on an ordered pair of properties consisting of one office building and one industrial building cannot be completely independent of the vector-valued pair of returns for the same time period on another office building and another industrial building. However, if intraclass return correlations are small, then most property return variance is independent of the returns on other properties of the same type. This suggests that, for purposes of sampling robust statistics such as the correlation function, the series of returns on the second pair of buildings is an acceptable proxy for a random sample independent of the first sample.

14 This is not the maximum that can be done to reduce the uncertainty of correlation estimates when the number of independent sample sets of *f* and *g* are unequal. Clearly, in the case of the example, nine independent sets of office building returns coupled with four independent sets of industrial building returns should provide more information about the correlation between office and industrial returns than four independent sets of returns for each property type—and consequently, reduce the width of a 95% confidence interval for the correlation. However, there is a problem in extracting that extra information due to the fact that a correlation estimator is a random variable that is a function of equal numbers of independent samples for both random variables. In the case of unequal numbers of sample sets for the two variables, it is necessary to dispense with elementary methods and turn to a maximum likelihood z-transform estimator conditioned on all available sample sets for the two random variables. While this is possible in principle (at least, under the simplifying assumption that the z-transform is normally distributed), in practice the computation of the information matrix represents an exercise in numerical applied mathematics that may not even be feasible, due to the need to numerically integrate a matrix function of an equal number of samples from each variable with respect to nonlinear functions of some of the variables in order to average away the effect of the missing samples.

The normalizing and variance-stabilizing properties of the z-transform apply only in the case of correlations between random variables that are real-valued. If either random variable is vector-valued, then the z-transform of the correlation usually is not normally distributed and its variance is not asymptotically independent of the true correlation value.[15]

An application of the z-transformation settles the question of whether a set of sample correlations are identical for statistical purposes. Since for all practical purposes the z-transformation of each correlation function can be assumed normally distributed, the sample z-transformations can be compared via one-way analysis of normally distributed random samples. The bias term in the z-transformation can be ignored, because the null hypothesis of the test assumes that the true correlation z-transformations are all identical, in which case the bias adjustments to all estimators would be identical, and hence irrelevant to the test statistic.

Data Description

Appraisal-based returns are generated by institutional investment managers for internal use and to meet institutional client reporting requirements, and usually are not reported to the general public. However, in order to facilitate the generation of aggregate real estate performance statistics needed by institutional investors, quarterly returns together with supporting descriptive and analytical data have been reported to NCREIF by member investment managers and some pension plan sponsors since 1978, and assembled into a commercial real estate investment performance data base. Aggregate statistics of the data base have been published on a regular basis by NCREIF since the early 1980s. These data releases constitute the largest set of consistently reported, institutional-grade property return data for the United States available to academic researchers.

If ownership of a data base property is transferred between two NCREIF members, and the new owner reports returns on that property to the data base, then after the transferal the property will be regarded as a new asset for purposes of NCREIF data base maintenance. Thus, while individual properties could remain within the data base for indefinitely long periods, individual return time series covering more than ten-year periods will likely remain uncommon unless NCREIF members change the current reporting policy. This poses an unavoidable burden upon researchers and practitioners seeking long return series.

This study uses annual sample correlations constructed from appraisal-based total return series for individual unleveraged properties in the NCREIF data base.[16] Discrete returns are used throughout, not continuously compounded return equivalents, because the emphasis of the study is on correlations and their implications for portfolio management rather than on the returns themselves.

---

15 A familiar example in which this breakdown occurs is for the case of the distribution of the multiple correlation coefficient R (i.e., the square root of R^2) that measures the degree of linear dependence between a real-valued random variable and two or more regressor variables. See Johnson & Kotz [1970, chap. 32.9] and Kendall, Stuart & Ord [1991, chap. 27].

16 The NCREIF data base contains individual property data on quarterly net operating income, capital improvement expenditures, and beginning and ending quarterly values. Pre-debt returns are computed for all properties. Prior to 1983, the data base only contained returns on unleveraged properties and in subsequent years the percentage of leveraged properties has grown slowly to approximately 20% of the total data base. Since deleveraged returns could only have contributed to exhibits based on return series beginning in 1983 or later (i.e., Exhibit 2), the authors decided to maintain consistency by dealing solely with unleveraged properties.

We examined correlations between time series of eight through eleven years duration.[17] The upper limit was set at eleven years because there are too few return time series for individual properties of greater length to test whether correlations cluster by property type and/or geographic region. The lower limit was set at eight years, because for time series with less than ten terms the statistics literature suggests that higher-order bias in our simple estimators of the z-transform mean and variance together with increasing deviation of the z-transform distribution from normality may be large enough to affect the accuracy of our numerical analyses in some situations (e.g., see Fisher [1921]; Gayen [1951]; Johnson & Kotz [1970]; Kendall, Stuart & Ord [1991]; and Pearson [1929]).[18]

Because we sought to examine differences among properties of the same or dissimilar type within the smallest available geographic area—the Metropolitan Statistical Area (MSA)—the data universe was further limited by NCREIF masking criteria to a maximum of seventeen MSAs and a minimum of four properties per MSA.

Real Estate Return Correlations

The limited length of annual sample return time series implies that large sample correlation values may occur without implying strong economic relations between investment returns on the underlying assets. For example, it is easily seen from the z-transform in the case of time series of eleven years duration that, for a sample correlation r_0=0.55, the 95% confidence interval for the true correlation is (–0.06, 0.86). Similarly, in the case of an annual return time series of eight years duration and a sample correlation r_1=0.65, the 95% confidence interval for the true correlation is (–0.07, 0.93). In other words, without additional statistical evidence it is impossible

17 In contrast to analyses of stock market return series, there is no significant possibility of survivorship bias in the data. All properties are unleveraged, are held in multi-property real estate portfolios that generate adequate cash flow to cover operating expenses, and are owned directly or beneficially by deep-pocketed investors. Thus, there is no possibility of operating insolvency or forced liquidation to satisfy legal obligations, which implies that asset sales occur almost solely on a planned and measured basis in response to portfolio adjustment decisions by investors or portfolio managers.

18 For each given sample series length, higher-order bias in our single-sample estimator for the z-transform mean increases as the absolute value of the true correlation increases. Similarly, for each given true correlation, higher-order bias in our estimator for the z-transform mean increases as the length of the sample series is reduced. Since the analysis in the next section suggests 0.2 as a close approximation to the true real estate product-moment correlation values, we tested the total bias of our estimator with a series of five Monte Carlo simulations of 20,000 correlation samples each from a bivariate normal distribution (for N=5,…,9) in which the true correlation between the two real-valued components was set equal to 0.2. Surprisingly, in each case the results indicated that the simple estimator is extremely accurate within the correlation range of interest. For example, for N=9, the mean Monte Carlo z-transform value was 0.216, compared with a first-order bias-adjusted value of 0.215 for the z-transform of the true correlation; and for N=5, the mean Monte Carlo z-transform value was 0.233, compared with a first-order bias-adjusted value of 0.228 for the z-transform of the true correlation. We also found our one-sample estimator for the z-transform standard deviation to be very accurate in all five cases: for example, for N=9, our estimator yielded a value for the z-transform standard deviation of σ=0.397, compared with a Monte Carlo estimate of 0.403; and for N=5, our estimator yielded a value for the z-transform standard deviation of σ=0.655, compared with a Monte Carlo estimate of 0.685. However, the Monte Carlo distributions also confirmed assertions in the statistics literature that the z-transform, while approximately normal, becomes progressively less so as N falls below ten: for example, for N=9, the kurtosis of the Monte Carlo distribution was 0.335; and for N=5, the Monte Carlo distribution kurtosis was 0.786 (skewness was negligible in all five cases), cf. Notes 10 and 11.

to conclude that either of these sample correlations implies a true correlation value greater than 0.00.[19]

Of course, analysts usually have available several independent correlation samples between groups of properties of similar type, in which case the widths of correlation confidence intervals are reduced. The problem is that analysts usually overstate the number of independent correlation samples available. For example, as discussed in Section 2, a correlation function based on time series of length *N* is a function of *2N* parameters, so that another independent sample of all *2N* parameters is required to generate an additional independent correlation sample. Thus, the maximum number of independent correlation samples that can be generated with eight series of office returns and three series of retail returns is three, not the twenty-four that researchers usually assume.

Our model for real estate returns assumes that the joint return distribution for each pair of properties is stationary and bivariate normal.[20] In addition, for each investment universe of real estate assets *S* that we examine, we assume that there is a finite number of subclasses $S_1,\ldots,S_n$, and correlations ρ_{ij} for each $1 \le i, j \le n$, such that ρ_{ij} is the true correlation between the return on p_i and p_j for each pair of properties $p_i \in S_i$ and $p_j \in S_j$.

Exhibits 1-4 (exhibits have a table identified as "Part A" and a graph identified as "Part B") show the sample correlations between annual returns from the NCREIF data base over various intervals of eight to eleven years in duration for the four basic commercial property types: office, retail, warehouse, and R&D/office.[21] In each case, the z-transforms of the correlation samples are averaged to obtain the best estimate for the ztransform of the true correlation, the maximum independent sample size is computed and used to estimate the efficient 95% confidence interval for the z-transform of the true correlation, and the inverse z-transform is used to convert the z-estimate together with confidence interval back into an efficient correlation estimate and 95% confidence interval for the true correlation. The exhibits also display the computed 95% confidence intervals in graphical form in Part B.

While nearly all correlation estimates in the exhibits are positive, in the majority of cases the 95% confidence interval includes zero, in which case the correlation estimates do not differ statistically from zero, as can be seen easily from Part B of the various exhibits.

---

[19] By contrast, correlations between stock and bond returns are calculated typically from monthly returns over five-year periods, in which case N=60. For example, in the case of a sample correlation of r_0=0.55, this implies a 95% correlation confidence interval of (0.34, 0.71). Although this example suggests that consequences of correlation uncertainties for MPT applications to stock and bond portfolio selection are less severe than for MPT applications to real estate, it also suggests the existence of pragmatic limitations to the accuracy—and hence value—of MPT stock and bond portfolio selection models which portfolio strategists typically do not take into account. See Graff, Young & Schoenberg [1995a,b].

[20] The assumption of bivariate normality is made only to ensure that the z-transform of the return correlation function is (approximately) normally distributed for reasonably small sample sizes. Since the correlation function is robust to perturbations in the shape of the joint return distribution, the restriction to normal returns can be relaxed considerably.

[21] Any change in the sample period can produce major changes in observed correlation samples. This is not just due to the effect of changes in each sample return series, but also to the fact that the subset of NCREIF data base properties used in computing correlation samples during each period can change completely by the slightest shift in the sample period. Since all existing MPT institutional portfolio strategies were based on much smaller property data bases than the one used in this study, it also follows that sample errors for correlations used in those strategies were substantially larger than fluctuations among the correlations in the exhibits.

Our main objective in this section is to test whether the correlations ρ_{ij} in Part A of each exhibit are identical, or more precisely whether or not the differences in the sample values are sufficient to conclude statistically that the true correlation values are distinct. The null hypothesis is that all ten true correlation values are equal (or all eight correlations, in the case of Exhibit 7).

Because the ten correlation z-estimators in each case are essentially normal, the proposition that the true values are all equal can be tested by using the fact that, when it is true,

$$\sum w_i(\bar{z}_i - \bar{z}_{av})^2$$

is distributed as χ^2 on nine degrees freedom, where each w_i is given by the reciprocal of the variance of $\bar{z}_i$ and $\bar{z}_{av}$ is the weighted average of $\bar{z}_i$ (weighted by the w_i). In the case of z-transformed correlations, the reciprocal of each variance is given by $(N - 8/3)\times$(maximum number of independent z's), where N is the number of years covered by the time series samples.

The average value of $\bar{z}$ (i.e., $\bar{z}_{av}$) in the last row in Part A of each exhibit is the average of the $\bar{z}$ values, weighted by the maximum number of independent samples for each $\bar{z}_i$. We use the $\bar{z}_{av}$ value as input in computing the contribution of each $\bar{z}_i$ to the χ^2 statistic. The last column of Part A of each exhibit shows these contributions, with the total χ^2 value in the last row of the column.

The largest χ^2 value for Exhibits 1-4 is 7.00, which is less than the 0.05 significance level of 16.92 for the χ^2 function with nine degrees of freedom. Thus, the result of each of the four χ^2 tests is that, at the 0.05 significance level, we cannot reject the hypothesis that the true correlations are all identical.[22]

Since the χ^2 test statistics in each table are consistent with the assumption that the individual sample correlations are identical, the best estimate for the true correlation from each data set is obtained from $\bar{z}_{av}$ via the inverse z-transform. Confidence intervals are obtained by estimating the maximum number of independent correlation samples in the same manner as the maximum number of independent correlations between properties of the same type (cf. Section 2). The resulting values are presented on the last line of Part A of each exhibit.

Exhibits 5 and 6 present the results of the same analysis, but across four MSAs rather than across four property types (each of these exhibits is restricted to a single property type). The conclusions to be drawn from these exhibits are the same as the conclusions of the analyses presented in the first four exhibits. While a couple of the sample correlations in Exhibit 5 are large (office-office correlations within Denver and within Phoenix), it is important to note that the number of independent correlation samples available in each case is only two, and the χ^2 test statistic for this exhibit is consistent with the assumption that there is only one true correlation underlying all these samples. This conclusion is supported by the sample intracity warehouse-warehouse correlations in Exhibit 6, which similarly do not suggest the existence of any common economic structure driving a significant portion of the investment returns on property of the same type within the same MSA.

Exhibit 7 presents the intraclass correlations for all property types across eight MSAs. Using the 0.05 significance level of 14.07 for χ^2 with seven degrees of freedom, the χ^2 test confirms the hypothesis that the true correlations are identical at the 0.05 significance level.

---

22 We do not assert that the test statistics of Exhibits 1-4 are strictly independent. However, the changes in the actual properties sampled together with wide fluctuations in the correlation sample values as the sample period varies suggest that the four χ^2 statistics represent distinct tests.

Although the individual estimates from Exhibits 1a-7a for the common correlation value ρ are not independent, the best estimate for ρ from the combined exhibits is obtained from the average of the estimates for $\overline{z_{av}}$ from Exhibits 1a-7a, weighted by the reciprocals of the variances associated with the estimates in the individual exhibits. This combined weighted estimate for ρ equals 0.203.

We do not assert that all true correlations are necessarily equal. However, due to the inability to detect statistical evidence of differences in true correlations between returns from individual properties, we assert that it is rational for sophisticated investors to develop portfolio strategies consistent with the assumption that all true correlations between individual annual property returns are equal and that this common correlation value is much smaller than typical correlations between monthly stock and bond returns.

In a recent article, Louargand [1992] presents the results of a survey of more than 400 of the largest institutional portfolio managers. Although a majority of these investors were cognizant of MPT mean-variance models, Louargand reports that only a small fraction make use of MPT models for portfolio management. Our results imply that this is rational investment behavior by sophisticated market participants, due to (1) the inability of portfolio strategists to discern differences between the risk reduction effectiveness of MPT mean-variance models and naive diversification, (2) the high degree of effectiveness suggested for the naive (low-tech) diversification strategy by the low sample values observed for real estate correlations, and (3) the additional costs in time and money associated with procuring market information necessary to implement MPT models.[23] Louargand also reports that most institutional portfolio managers acknowledge the risk reduction effectiveness of real estate portfolio diversification and make some systematic effort to diversify within the real estate asset class. In view of the relatively low ρ estimate obtained in this section, this also is consistent with the evidence of historical returns and can be regarded as rational investment behavior on the part of sophisticated investors.

Conclusions

In applications of MPT to efficient real estate portfolio diversification, it is necessary for investment purposes to know how real estate assets group. At the least, it is necessary to be able to decompose the real estate investment universe into a finite number of subgroups, such that investment returns are relatively homogeneous for assets *within* each group, with returns relatively heterogeneous *across* asset groups.

In order for an investment universe decomposition model to meet these criteria, it is both necessary and sufficient that intraclass correlations be higher statistically than interclass correlations. If this condition is not satisfied, then diversification via the model is no more efficient from an *ex ante* perspective than the naive portfolio selection strategy of diversifying without attention to how individual assets distribute across the model subgroups.

The χ^2 tests for Exhibits 1-4 show that appraisal-based NCREIF annual returns series do not support the contention that diversification across property type is more efficient than naive diversification. Similarly, the χ^2 tests of Exhibits 5 and 6 show that appraisal-based annual

---

[23] Cole, *et al.* [1989] observed that the incremental information cost associated with MPT portfolio selection is greater than immediately apparent, because portfolio managers must also acquire the local real estate market data needed for cost-effective property management of assets selected via MPT methodology.

returns series do not support the contention that, within property type, diversification across MSA is more efficient than naive diversification.

The exhibits also show that sample correlations are extremely unstable, and that the 95% confidence intervals around correlation samples are very large. In fact, it is the large size of these confidence intervals that is responsible for the failure of the χ^2 tests to distinguish statistically between the sample correlations.

The large width of the confidence intervals offers an explanation for the large observed fluctuations in real estate correlation samples across time periods that is consistent with the assumption of stationary return distributions. Absent wide confidence intervals, sample correlations would imply conclusively that real estate returns are not stationary, in which case historical returns would be useless as input in forecasting future investment performance.

The principle underlying this analysis is intuitive: correlation estimates based on short time series are subject to large sample errors (even with multiple samples from each time period and cross-sectional techniques to refine the estimates) which translate into large uncertainties in quantitative portfolio diversification strategies. Because NCREIF return time series for individual properties terminate when property ownership changes (a consequence of its data masking policy), few NCREIF time series are likely to include more than fifteen years of returns. Thus, unless NCREIF relaxes its data masking policy, the limits determined by our analysis will continue to be valid without modification for decades to come.

Although NCREIF does not presently report disaggregated data broken down by so-called economic regions, our examination of breakdown by MSA (a more fine-grained geographic and economic boundary) in Exhibit 7 leads us to conclude that this diversification model would receive no more support from the methodology of this article than other MPT diversification models have received.

The correlation estimates in the exhibits suggest that true correlations among individual properties of various types are smaller than typical correlations among stocks. It follows that portfolio managers are rational to believe that the addition of properties of any type to portfolios with small numbers of individual assets is an effective diversification strategy, and to be skeptical of the additional value of MPT mean-variance diversification strategies.

This is not meant as an assertion that diversification across property type and/or geographical region is without incremental risk reduction potential relative to diversification within the same property type and geographical region, but rather that diversification across property type and geographical region must still be made on intuitive rather than formulaic grounds. The professional real estate investor must decide—based on experience and common sense rather than correlations and a quantitative optimization model—whether the incremental costs of expanding a commercial real estate investment portfolio beyond the investor's professional and geographical domains of expertise are justified by the perceived reduction in the level of intuitive investment risk. The state-of-the-art in quantitative real estate portfolio strategy is simply not yet good enough to yield more precise conclusions. Absent major improvements in the quality and quantity of available data on individual property investment returns and/or portfolio modeling, real estate portfolio strategy must remain more art than science.

References

Chesire, Leona, Elena Oldis, and Egon S. Pearson (1932). “Further Experiments on the Sampling Distribution of the Correlation Coefficient,” *Journal of the American Statistical Association* 27, 121-128.

Cole, Rebel, David Guilkey, Mike Miles, and Brian Webb (1989). “More Scientific Diversification Strategies for Commercial Real Estate,” *Real Estate Review* 19, 59-66.

Efron, Bradley and Robert J. Tibshirani (1986). “Bootstrap Methods for Standard Errors, Confidence Intervals, and Other Methods of Statistical Accuracy,” *Statistical Science* 1, 54-77.

Fisher, Ronald A. (1915). “Frequency Distributions of the Values of the Correlation Coefficient in Samples from an Indefinitely Large Population,” *Biometrika* 10, 507-521.

Fisher, Ronald A. (1921). “On the 'Probable Error' of a Coefficient of Correlation Deduced from a Small Sample,” *Metron* 1, 3-32.

Gayen, A.K. (1951). “The Frequency Distribution of the Product-Moment Correlation Coefficient in Random Samples of Any Size Drawn From Non-Normal Universes,” *Biometrika* 38, 219-247.

Geltner, David M. (1993). “Temporal Aggregation in Real Estate Return Indices,” *Journal of the American Real Estate and Urban Economics Association* 21, 141-166.

Geltner, David M., Richard A. Graff, and Michael S. Young (1994). “Random Disaggregate Appraisal Error in Commercial Property: Evidence from the Russell-NCREIF Database,” *Journal of Real Estate Research* 9, 403-419.

Giliberto, S. Michael. (1990). “Equity Real Estate Investment Trusts and Real Estate Returns,” *Journal of Real Estate Research* 5, 259-264.

Graff, Richard A. (1995). “The Impact of Annual Real Estate Appraisals on Sample Correlations Computed from Quarterly Returns,” Working paper presented at American Real Estate Society annual meeting, Hilton Head Island, SC.

Graff, Richard A., Michael S. Young, and Ronald J. Schoenberg (1995a). “Limitations on the Accuracy of Markowitz Optimization: The Efficient Frontier as a Family of Sample Statistics,” Working paper presented at American Real Estate Society annual meeting, Hilton Head Island, SC.

Graff, Richard A., Michael S. Young, and Ronald J. Schoenberg (1995b). “Mean-Variance Efficient Portfolios: Fool’s Gold in the Investment Strategy Treasure Chest,” Working paper.

Gyourko, Joseph and Donald Keim (1992). “What Does the Stock Market Tell Us About Real Estate Returns?” *Journal of the American Real Estate and Urban Economics Association* 20, 457-485.

Hartzell, David J., John Hekman, and Mike Miles (1986). “Diversification Categories in Investment Real Estate,” *Journal of the American Real Estate and Urban Economics Association* 14, 230-254.

Hartzell, David J., David G. Shulman, and Charles H. Wurtzebach (1987). “Refining the Analysis of Regional Diversification for Income-Producing Real Estate,” *Journal of Real Estate Research* 2, 85-95.

Hotelling, Harold (1953). “New light on the Correlation Coefficient and its Transforms” (with discussion), *Journal of the Royal Statistical Society*, Series B, 15, 193-232.

Johnson, Norman L. and Samuel Kotz (1970). *Continuous Univariate Distributions, Volume 2*, Boston: Houghton Mifflin.

Kendall, Maurice G., Alan Stuart, and J.Keith Ord (1991). *Kendall's Advanced Theory of Statistics, Volume 2,* New York: Oxford University Press, Fifth Edition.

Levy, Haim and Harry M. Markowitz (1979). "Approximating Expected Utility by a Function of Mean and Variance," *American Economic Review* 69, 308-317.

Liang, Youguo, F.C.Neil Myer, and James R. Webb (1994). "The Bootstrap Efficient Frontier for Mixed-Asset Portfolios," *Journal of Real Estate Economics*, forthcoming.

Liu, Crocker H., David J. Hartzell, and Terry V. Grissom (1992). "The Role of Co-Skewness in the Pricing of Real Estate," *Journal of Real Estate Finance and Economics* 5, 299-319.

Lin, Ching-Chuong and Govind S. Mudholkar (1980). "A Simple Test for Normality Against Asymmetric Alternatives," *Biometrika* 67, 455-461.

Louargand, Marc A. (1992). "A Survey of Pension Fund Real Estate Portfolio Risk Management Practices," *Journal of Real Estate Research* 7, 361-373.

Miles, Mike and Thomas McCue (1984a). "Diversification in the Real Estate Portfolio," *Journal of Financial Research* 7, 57-68.

Miles, Mike and Thomas McCue (1984b). "Commercial Real Estate Returns," *Journal of the American Real Estate and Urban Economics Association* 12, 355-377.

Myer, F.C.Neil and James R. Webb (1994). "Statistical Properties of Returns: Financial Assets Versus Commercial Real Estate," *Journal of Real Estate Finance and Economics* 8, 267-282.

Pearson, Egon S. (1929). "Some Notes on Sampling Tests with Two Variables," *Biometrika* 21, 337-360.

Wheaton, William C. and Raymond G. Torto (1989). "Income and Appraised Values: A Reexamination of the FRC Returns Data," *Journal of the American Real Estate and Urban Economics Association* 17, 439-449.

Young, Michael S. and Richard A. Graff (1995). "Real Estate is Not Normal: A Fresh Look at Real Estate Return Distributions," *Journal of Real Estate Finance and Economics* 10, 225-259.

Young, Michael S. and D. Wylie Greig (1993). "Drums Along the Efficient Frontier," *Real Estate Review* 22, 18-29.

Exhibit 1a
Correlation of Total Annual Returns with Fisher's z-transform
NCREIF Data for the 8-year Period 1980 to 1987

Office (O)	24
Retail (R)	12
Warehouse (W)	61
R&D (D)	16
Total	113 properties

	$\bar{z}$	bias in z	$\bar{z}$	r_0	r_l	r_u	number of z's	max. no. indep z's	χ^2
O vs O	0.281	0.020	0.261	0.256	0.016	0.467	276	12	3.11
O vs R	-0.034	-0.002	-0.032	-0.032	-0.270	0.210	288	12	0.57
O vs W	0.095	0.007	0.088	0.088	-0.085	0.256	1,464	24	0.15
O vs D	0.070	0.005	0.065	0.065	-0.146	0.270	384	16	0.01
R vs R	0.010	0.001	0.009	0.009	-0.325	0.341	66	6	0.08
R vs W	0.019	0.001	0.018	0.018	-0.224	0.257	732	12	0.11
R vs D	-0.003	0.000	-0.003	-0.003	-0.243	0.238	192	12	0.26
W vs W	0.044	0.003	0.041	0.041	-0.114	0.193	1,830	30	0.04
W vs D	0.063	0.004	0.059	0.058	-0.152	0.264	976	16	0.00
D vs D	0.005	0.000	0.005	0.005	-0.287	0.296	120	8	0.13
Total	0.061	0.004	0.056	0.056	-0.057	0.168	6,328	56	4.47

Exhibit 1b
95% Confidence Intervals of Correlation Coefficients
NCREIF Data for the 8-year Period 1980 to 1987

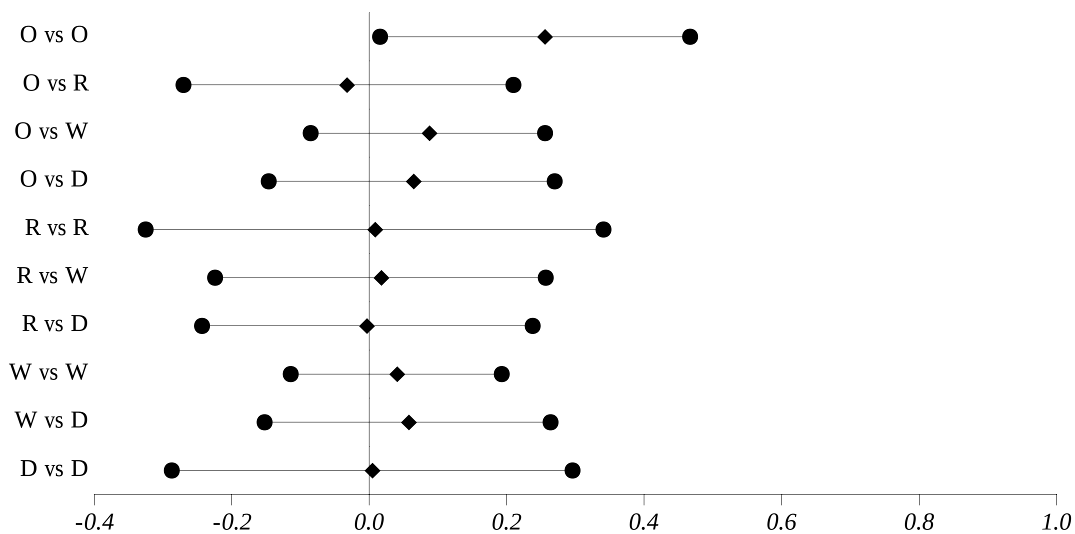

Exhibit 2a
Correlation of Total Annual Returns with Fisher's z-transform
NCREIF Data for the 8-year Period 1984 to 1991

Office (O)	56
Retail (R)	23
Warehouse (W)	74
R&D (D)	25
Total	178 properties

	$\bar{z}$	bias in z	$\bar{z}$	r_0	r_l	r_u	number of z's	max. no. indep z's	χ^2
O vs O	0.319	0.022	0.297	0.289	0.136	0.428	1,540	28	0.08
O vs R	0.290	0.020	0.270	0.263	0.093	0.419	1,288	23	0.00
O vs W	0.263	0.018	0.245	0.240	0.130	0.343	4,144	56	0.32
O vs D	0.301	0.021	0.280	0.273	0.110	0.422	1,400	25	0.00
R vs R	0.357	0.024	0.333	0.321	0.076	0.529	253	11	0.22
R vs W	0.288	0.020	0.268	0.262	0.091	0.418	1,702	23	0.01
R vs D	0.351	0.024	0.327	0.316	0.149	0.465	575	23	0.38
W vs W	0.271	0.019	0.252	0.247	0.112	0.373	2,701	37	0.12
W vs D	0.295	0.020	0.275	0.268	0.104	0.417	1,850	25	0.00
D vs D	0.324	0.022	0.302	0.293	0.057	0.498	300	12	0.05
Total	0.296	0.021	0.275	0.268	0.183	0.350	15,753	89	1.18

Exhibit 2b
95% Confidence Intervals of Correlation Coefficients
NCREIF Data for the 8-year Period 1984 to 1991

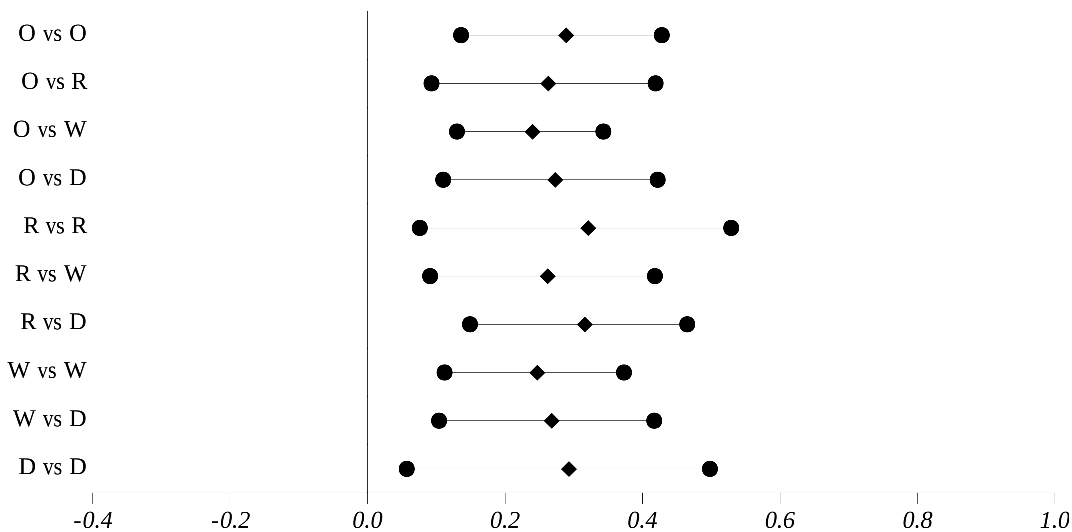

Exhibit 3a
Correlation of Total Annual Returns with Fisher's z-transform
NCREIF Data for the 9-year Period 1982 to 1990

Office (O)	44
Retail (R)	18
Warehouse (W)	66
R&D (D)	21
Total	149 properties

	$\bar{z}$	bias in z	$\bar{z}$	r_0	r_l	r_u	number of z's	max. no. indep z's	χ^2
O vs O	0.313	0.019	0.294	0.286	0.127	0.430	946	22	3.58
O vs R	0.165	0.010	0.155	0.154	-0.029	0.326	792	18	0.02
O vs W	0.100	0.006	0.094	0.093	-0.024	0.208	2,904	44	0.78
O vs D	0.190	0.012	0.178	0.176	0.008	0.335	924	21	0.18
R vs R	0.219	0.013	0.206	0.203	-0.054	0.434	153	9	0.25
R vs W	0.091	0.006	0.085	0.085	-0.098	0.263	1,188	18	0.44
R vs D	0.210	0.013	0.197	0.195	0.013	0.363	378	18	0.37
W vs W	0.098	0.006	0.092	0.092	-0.044	0.224	2,145	33	0.63
W vs D	0.107	0.007	0.100	0.100	-0.070	0.264	1,386	21	0.28
D vs D	0.158	0.010	0.148	0.147	-0.098	0.375	210	10	0.00
Total	0.153	0.009	0.143	0.142	0.053	0.230	11,026	74	6.52

Exhibit 3b
95% Confidence Intervals of Correlation Coefficients
NCREIF Data for the 9-year Period 1982 to 1990

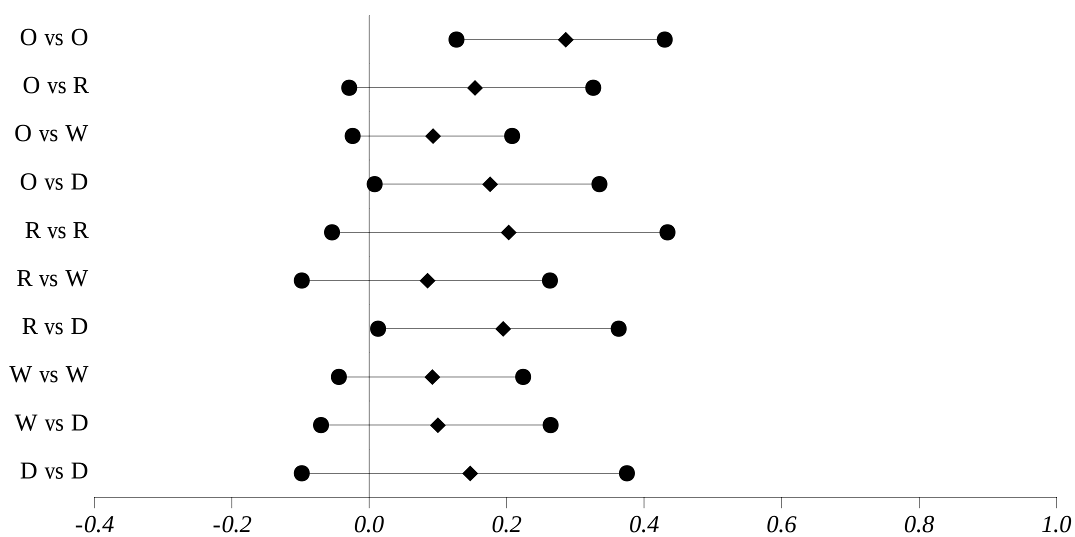

Exhibit 4a
Correlation of Total Annual Returns with Fisher's z-transform
NCREIF Data for the 11-year Period 1981 to 1991

Office (O)	20
Retail (R)	14
Warehouse (W)	50
R&D (D)	14
Total	98

	$\bar{z}$	bias in z	$\bar{z}$	r_0	r_l	r_u	number of z's	max. no. indep z's	χ^2
O vs O	0.459	0.021	0.438	0.412	0.219	0.573	190	10	1.86
O vs R	0.377	0.018	0.359	0.344	0.176	0.493	280	14	0.53
O vs W	0.279	0.014	0.265	0.259	0.113	0.395	1,000	20	0.16
O vs D	0.363	0.017	0.346	0.332	0.163	0.483	280	14	0.33
R vs R	0.458	0.021	0.437	0.411	0.178	0.600	91	7	1.28
R vs W	0.271	0.013	0.258	0.252	0.076	0.413	700	14	0.17
R vs D	0.348	0.017	0.331	0.320	0.149	0.472	196	14	0.17
W vs W	0.219	0.011	0.208	0.205	0.072	0.331	1,225	25	1.71
W vs D	0.232	0.011	0.221	0.217	0.039	0.382	700	14	0.70
D vs D	0.273	0.013	0.260	0.254	0.003	0.475	91	7	0.08
Total	0.310	0.015	0.295	0.286	0.195	0.373	4,753	49	7.00

Exhibit 4b
95% Confidence Intervals of Correlation Coefficients
NCREIF Data for the 11-year Period 1981 to 1991

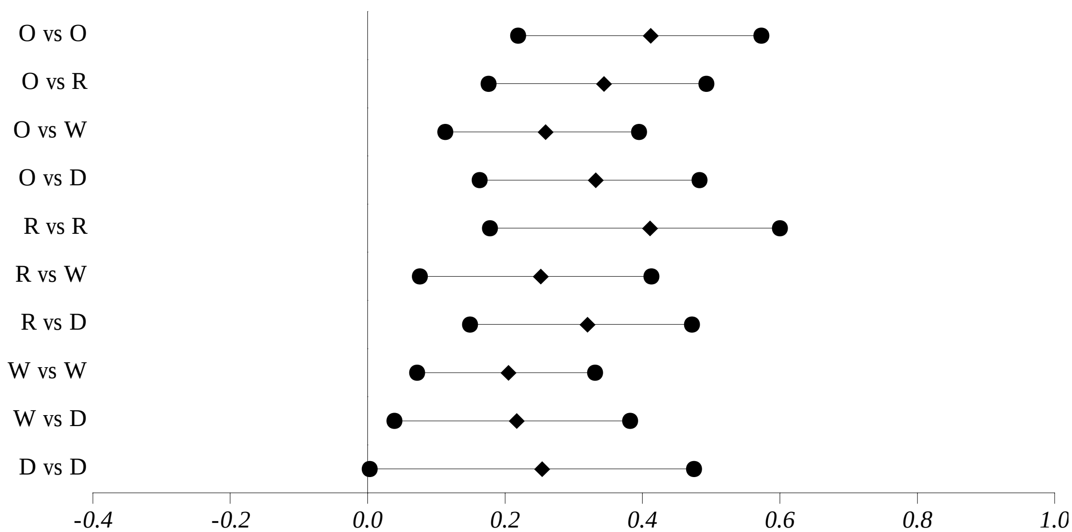

Exhibit 5a
Correlation of Total Annual Returns with Fisher's z-transform
NCREIF Data for the 9-year Period 1981 to 1989
Office Properties Only

Chicago (C)	4
Denver (D)	4
Phoenix (P)	4
Washington (W)	5
Total	17 properties

	$\bar{z}$	bias in z	$\bar{z}$	r_0	r_l	r_u	number of z's	max. no. indep z's	χ^2
C vs C	0.439	0.026	0.413	0.391	-0.137	0.746	6	2	0.00
C vs D	0.460	0.027	0.433	0.408	0.044	0.676	16	4	0.01
C vs P	0.482	0.028	0.454	0.425	0.065	0.688	16	4	0.05
C vs W	0.275	0.017	0.258	0.253	-0.130	0.570	20	4	0.67
D vs D	0.718	0.038	0.680	0.591	0.128	0.843	6	2	1.00
D vs P	0.663	0.036	0.627	0.556	0.233	0.768	16	4	1.29
D vs W	0.285	0.017	0.268	0.261	-0.121	0.576	20	4	0.59
P vs P	1.187	0.052	1.135	0.813	0.526	0.934	6	2	7.11
P vs W	0.029	0.002	0.027	0.027	-0.347	0.394	20	4	4.23
W vs W	0.271	0.017	0.254	0.249	-0.288	0.667	10	2	0.35
Total	0.438	0.026	0.412	0.390	0.136	0.596	136	8	15.30

Exhibit 5b
95% Confidence Intervals of Correlation Coefficients
NCREIF Data for the 9-year Period 1981 to 1989

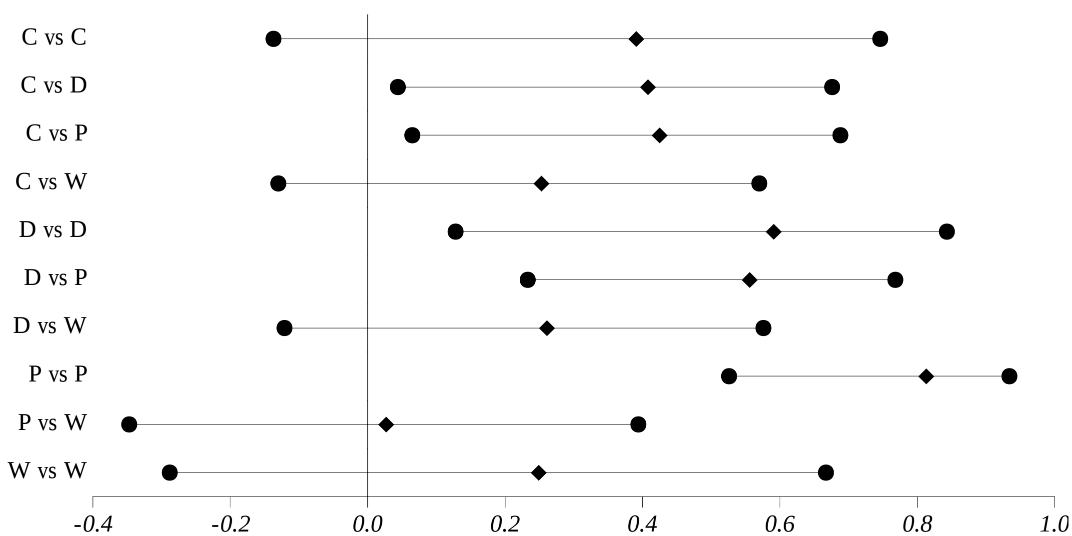

Exhibit 6a
Correlation of Total Annual Returns with Fisher's z-transform
NCREIF Data for the 10-year Period 1981 to 1990
Warehouse Properties Only

Anaheim (A)	7
Chicago (C)	7
Dallas (D)	5
Los Angeles (L)	8
Total	27 properties

	$\bar{z}$	bias in z	$\bar{z}$	r_0	r_l	r_u	number of z's	max. no. indep z's	χ^2
A vs A	0.062	0.003	0.059	0.058	-0.345	0.443	21	3	0.00
A vs C	0.064	0.004	0.060	0.060	-0.210	0.322	49	7	0.00
A vs D	0.068	0.004	0.064	0.064	-0.254	0.370	35	5	0.00
A vs L	0.007	0.000	0.007	0.007	-0.261	0.273	56	7	0.18
C vs C	0.172	0.009	0.163	0.161	-0.250	0.523	21	3	0.24
C vs D	0.036	0.002	0.034	0.034	-0.282	0.343	35	5	0.03
C vs L	0.151	0.008	0.143	0.142	-0.130	0.394	56	7	0.37
D vs D	0.267	0.014	0.253	0.247	-0.254	0.644	10	2	0.59
D vs L	-0.062	-0.003	-0.059	-0.058	-0.365	0.259	40	5	0.61
L vs L	0.049	0.003	0.046	0.046	-0.306	0.387	28	4	0.01
Total	0.067	0.004	0.063	0.063	-0.137	0.258	351	13	2.03

Exhibit 6b
95% Confidence Intervals of Correlation Coefficients
NCREIF Data for the 10-year Period 1981 to 1990
Warehouse Properties Only

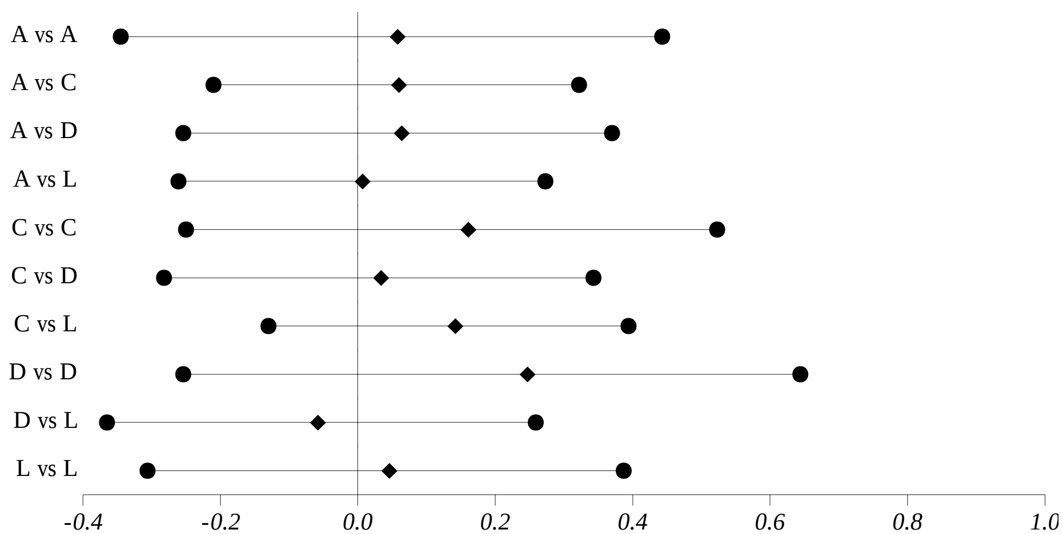

Exhibit 7a
Correlation of Total Annual Returns with Fisher's z-transform
NCREIF Data for the 10-year Period 1981 to 1990
Properties within Selected MSAs

Anaheim	9
Chicago	12
Dallas	8
Denver	8
Los Angeles	11
Minneapolis	9
Phoenix	8
Washington	11
Total	76 properties

	$\bar{z}$	bias in z	$\bar{z}$	r_0	r_l	r_u	number of z's	max. no. indep z's	χ^2
Anaheim	0.089	0.005	0.084	0.084	-0.271	0.419	36	4	0.77
Chicago	0.178	0.010	0.168	0.167	-0.127	0.433	66	6	0.23
Dallas	0.401	0.021	0.380	0.363	0.018	0.630	28	4	0.66
Denver	0.121	0.007	0.114	0.114	-0.243	0.443	28	4	0.49
Los Angeles	0.084	0.005	0.079	0.079	-0.240	0.383	55	5	1.02
Minneapolis	0.358	0.019	0.339	0.327	-0.023	0.605	36	4	0.34
Phoenix	0.247	0.013	0.234	0.229	-0.128	0.534	28	4	0.00
Washington	0.536	0.027	0.509	0.469	0.183	0.682	55	5	2.98
Total	0.251	0.014	0.237	0.233	0.116	0.343	332	36	6.50

Exhibit 7b
95% Confidence Intervals of Correlation Coefficients
NCREIF Data for the 10-year Period 1981 to 1990
Properties within Selected MSAs