Revisiting Non-Normal Real Estate Return Distributions by Property Type in the U.S.

by

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Abstract: In this updated empirical analysis, investment risk models with infinite variance are more descriptive of distributions of individual property returns in the NCREIF database over the period 1980 to 2003 than Normally distributed risk models. Real estate investment risk is heteroskedastic, but the Characteristic Exponent of the investment risk function is nearly constant across time although differences among property types are evident. Accordingly, asset diversification is far less effective at reducing the impact of non-systematic investment risk on real estate portfolios than in the case of assets with Normally distributed investment risk. The patterns found in the U.S. are the same in Australia and the United Kingdom, and the Characteristic Exponents are virtually identical across all three countries.

The data and analysis of this note extend the research presented by Young and Graff (1995) with corrected data from the 1980 to 1992 period; new data from the 1993 to 2003 period; comparisons among U.S., U.K., and Australian return distributions; and more discussion of implications of the findings. That earlier work presented an empirical analysis of the distributional characteristics of cross-sectional annual returns of individual assets contained within the NCREIF Property Index from 1980 through 1992, in the aggregate and disaggregated by property type. The authors found that cross-sectional annual returns were not Normally distributed during any calendar year studied. Additionally, the authors found that both skewness and the magnitude of real estate risk changed over time, i.e., was heteroskedastic.

Now, with the passage of time across a wider range of macroeconomic conditions, this work can be extended to 2003. For consistency and comparability, the methodology of this paper is identical to the 1995 study. Also, the identical methodology has been applied to Australian and United Kingdom institutional properties in Graff, Harrington, and Young (1997) and Young, Lee, and Devaney (2006) respectively. Thus, similarities in real estate return distributions can be assessed across three English-speaking countries.

As institutional real estate investors search the globe for investment opportunities, it can be helpful to understand the behavioral characteristics of assets that might be added to portfolios. If performance characteristics vary among real estate assets in different countries, these differences might lead to differences in portfolio strategies for the global investor. However, if there are similarities among performance characteristics, then investors may realize efficiencies by extending effective strategies from the home country to foreign soil.

To repeat, this paper tests empirically whether property return distributions have finite variance and are Gaussian Normal. The short answer is that they still are not.

Data Description

Individual property returns of institutional-grade U.S. commercial real estate are available from NCREIF. Furthermore, the NCREIF Property Index has various sub-indices compiled along major geographical region, property type, and combined region and property type dimensions.

Reported returns are based on income and asset value changes (i.e., capital gains) as determined by periodic valuation by investment managers or third-party appraisers. Both quarterly and annual returns are available, but we use only the annual total returns provided by NCREIF for the calendar years 1980 to 2003 disaggregated by three property types: Office, Retail, and Industrial. By using annual returns we avoid the potential criticism that the quarterly returns are autocorrelated or that valuations are stale in the sense that valuations are not conducted quarterly on each property.[1]

Over the last several years, NCREIF has made a concerted effort to correct errors and misspecifications in the historic database. Although the specific changes are not available to the public, a comparison of the return distributions reported in the 1995 study with the current one reveal that changes affected shape statistics in particular years in the aggregate and across property types, but the overall average changes for the 1980 to 1992 period are relatively small. Exhibit 3 shows the differences in the yearly estimates of the characteristic exponent between the 1995 study and the current study.

Real Estate Return Model

A comparison of the data in the NCREIF Property Type sub-indices reveals significant differences among their annual returns. Thus, our real estate model presumes that expected variations in annual property returns due to differences in property type account for all of the differences in returns on properties.[2]

In model terms, the observed annual total return on each commercial property *p* during the calendar year *t* is of the following form:

$$R_t(p) = \mu_t(h(p)) + \varepsilon_t(p) \qquad (1)$$

where $h()$ is the property type (office, retail, or industrial), $\mu_t()$ is the expected total return during year *t* as a function of property type, and $\varepsilon_t(p)$ is a stable (possibly, infinite-variance) random variable. In addition, the model presumes that, for each $t \geq 1980$, the $\varepsilon_t()$ are independent identically distributed random variables with Characteristic Exponent $\alpha_t > 1.0$ and zero mean, and that $\varepsilon_{t_1}(p_i)$ and $\varepsilon_{t_2}(p_j)$ are independent for all $t_1 \neq t_2$ and all *i* and *j*.[3]

Under these assumptions, the random variable $\varepsilon_t(p)$ corresponds to the asset-specific investment risk of property p during period *t*, while the systematic and market sector real estate risk is described by the function $\mu_t(h(\ ))$.

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[1] Before beginning the data analysis, each discrete annual sample return r_t in the NCREIF database has been replaced with its continuously compounded logarithmic equivalent. Only properties having four quarters of data in a given calendar year have been included.

[2] Alternatively, we could have disaggregated returns by major geographic region. However, property type is probably the superior cut, because it is more likely that investment characteristics of commercial property differ for properties with different drivers of economic performance than for properties with the same economic and functional attributes situated in different parts of the country. The free flow of institutional real estate investment capital across the U.S. over the past thirty years tends to homogenize transient differences in investment characteristics across geographical regions for property of the same type.

[3] The assumption that $\alpha_t > 1.0$ guarantees that the mean of $\varepsilon_t(p)$ exists.

Tests and Results

Exhibit 1a shows the distributions of continuously compounded annual total returns for the years 1980 to 2003 in the aggregate. Superimposed upon the sample histogram is the Normal density function with the identical mean and standard deviation.[4] The distribution takes on a shape that is virtually identical across all property types and indeed virtually identical across national borders as will be discussed later. In particular, the sample density function is more peaked near the mean than the corresponding Normal density, has weaker shoulders and fatter tails (i.e., is leptokurtotic), and is negatively skewed. These distinctions are more apparent in the graph of the differences between the sample density and the Normal density in Exhibit 1b.

McCulloch's (1986) quantile methodology that employs a series of tables that must be interpolated doubly to determine parameter estimates was used to fit a stable distribution to each set of residuals ordered by calendar year and property type.[5] To test whether the parameters varied during the sample period, stable parameters were estimated for sets composed of the residuals aggregated across all years and property types. These results are tabulated in Exhibit 2 and are displayed graphically together with one and two standard deviation error bands in Exhibits 4 to 6 for the stable distribution parameters α, β, and γ (δ is irrelevant because the Location Parameter is an estimator for the mean and the analysis adjusts for the effect of varying means).

In the case of Characteristic Exponents α_t estimated by calendar year and property type, 94% (68 of 72) were distinct statistically from 2.0—the Characteristic Exponent of the Normal distribution—with 95% confidence and 82% (59 of 72) were distinct from 2.0 with 99% confidence. In the case of residuals aggregated across property type (the first panel of Exhibit 2), all twenty-four sample Characteristic Exponents α_t were distinct from 2.0 with 99% confidence.[6]

In the case of the Skewness Parameter β_t for all residuals aggregated across property type, 79% (19 of 24) were statistically significant (i.e., non-zero) with 99% confidence, and one remaining sample value was significant with 95% confidence. Furthermore, negative skewness results outnumber positive skewness results 17 to 7 times.

Exhibit 4 displays the sample Characteristic Exponents α_t of both the aggregated and individual property type residuals. It appears that α_t could be time-invariant. However, Exhibit 6 that shows graphical representations of these data, suggests that α_t likely varies across property type. From Exhibit 2, for the entire 1980 to 2003 sample period, estimates of Characteristic Exponents together with their

4 There are 51 "bins" in the histogram that span the range from minus to plus five standard deviations. Because some samples extend beyond this range, all the samples beyond plus or minus five standard deviations are included in the two extreme bins.

5 Since publication of McCulloch's technique, maximum likelihood estimation (MLE) has gained favor for parameter estimation. In a test of MLE versus McCulloch on the aggregate NCREIF data set, the MLE results from the Mathematica modeling software were virtually identical to those produced by interpolation, so little more than speed of computation was gained using MLE. Thus, this note uses the McCulloch technique for consistency and comparability with prior work.

6 Of the four parameters that describe the stable distribution, the Characteristic Exponent is considered the most helpful for expressing the shape of the distribution. The Characteristic Exponent α lies in the half-open interval (0,2] and measures the rate at which the tails of the density function decline to zero. The larger the value of the Characteristic Exponent α, the faster the tails shrink toward zero. When α=2.0, the distribution is Normal. While the means (first moments) of stable distributions with Characteristic Exponents α>1.0 do exist, variances (second moments) do not exist—i.e., are infinite—for those distributions with Characteristic Exponents α<2.0.

standard errors are 1.434 ±0.011 for all three property types combined, 1.487 ±0.021 for Office properties, 1.337 ±0.022 for Retail properties, and 1.466 ±0.017 for Industrial properties.

By contrast, Exhibit 5 shows clearly that β_t is not time-invariant. Indeed, β_t for all properties displayed a roughly cyclic pattern throughout the test period (the results are indeterminate for individual property types due to the large widths of the error bands), but seem to track one another especially the Office and Industrial results.

Exhibit 6 shows clearly that the Scale Parameter γ is not time-invariant either in the aggregate or by property type. The general time-series patterns, however, are quite similar with roughly the same peaks and valleys. Since γ is the stable infinite-variance measure of risk, this means that asset-specific risk is heteroskedastic.

The three graphs of Exhibit 7 show the Characteristic Exponent, the Skewness, and the Scale Parameter for each property type and the aggregate over the full 1980 to 2003 time period along with the one- and two-sigma error bands. In the case of the Characteristic Exponent, Office and Industrial are statistically indistinguishable and Retail is the outlier. For the Skewness results, Retail and Industrial property types are statistically indistinguishable and Office and Retail show marginal overlap. In terms of Scale Parameter, all three property type results differ statistically from one another.

The above analysis implies that (1) real estate investment risk during the sample period was heteroskedastic; (2) during virtually all sample subperiods and across property type, stable infinite-variance skewed asset-specific risk functions with a Characteristic Exponent α of approximately 1.434 modeled the observed distributions of return residuals better than Normally distributed risk candidates; and (3) property type differences in the Characteristic Exponent across property types are likely, certainly Retail properties showed notable differences from Office and Industrial over the full 1980 to 2003 sample period.

Comparisons Among U.S., U.K., and Australian Property Returns

Following publication of the Young and Graff (1995) study, the authors conducted a study of Australian property return distributions using the same methodology, Graff, Harrington, and Young (1997). Despite the much smaller sample size in the Australian data set (4,593 versus 33,745 in the current U.S. set), the statistical findings were statistically indistinguishable from the U.S. findings. Comparing the Characteristic Exponent results of the earlier Australian study with the current updated U.S. study, the statistical estimate of Characteristic Exponent of U.S. returns together with a 95% confidence interval around the value is 1.434 ± 0.022 versus the estimate of Australian returns of 1.588 ± 0.068, not statistically identical as reported earlier when the U.S. estimate was somewhat higher with greater standard error.

Young, Lee, and Devaney (2006) examined U.K. property returns in the IPD database over the 1981 to 2003 period, again using the same methodology as Young and Graff (1995). The 269,853-property sample size of the U.K. data set dwarfs both the U.S. and Australian samples. The statistical equivalence of the Characteristic Exponent between the current U.S. study and the U.K. study is evident. The U.K. estimate with a 95% confidence interval is 1.448 ± 0.008 versus the U.S. estimate of 1.434 ± 0.022. By property type, the Office and Industrial properties in the U.S. and U.K. samples are statistically identical at 95% confidence with respect to Characteristic Exponent. Retail properties, however, show a somewhat contrary result relative to the Characteristic Exponent of the country aggregates: the Characteristic

Exponent of Retail properties in the U.S. sample is 1.337 ± 0.043 versus 1.471 ± 0.012 in the U.K. sample.

The stable distribution parameters and sample sizes of the current U.S., U.K., and Australian studies are shown in Exhibit 8.

Implications for Portfolio Management

To examine the impact on portfolio risk reduction in light of the distinctly non-normal distribution of real estate returns, the risk reduction formula for the Scale Parameter of a portfolio with Characteristic Exponent *a* and number of assets *n* would be:

$$\gamma_p = n^{(1/\alpha)-1}\gamma_f \qquad (2)$$

For any given α>1.0, the reduction in asset-specific risk increases with increasing n. As α diminishes to 1.0 from its upper limit of 2.0, the reduction in asset-specific risk likewise diminishes for any given n> 1. The degree to which asset-specific risk can be reduced is tied directly to the shape of the distribution. Portfolios whose return distributions have positive or negative tails fatter than portfolios whose return distributions are Gaussian Normal require more assets to produce the same risk-reduction, which is why significant departures from Normality should matter to those who assemble portfolios with the expressed intent to reduce asset-specific risk. For the actual NCREIF data of this study, the asset-specific risk reduction potential of portfolios can be compared to the theoretical portfolio with Normally distributed returns in two ways.

The sample value α=1.434 from the preceding section provides a practical estimate for the effect of portfolio diversification on asset-specific risk reduction:

$$\gamma_p \approx n^{-0.303}\gamma_f \qquad (2')$$

For example, a typical U.S. closed-end real estate fund or client separate account might have 10 to 20 properties, and large open-end real estate funds might have about 100 properties. With these assumptions, the magnitude of combined asset-specific risk for such a closed-end fund or client separate account is between 40% and 50% of the magnitude of asset-specific risk for a single property portfolio. However, if the asset-specific risk were Normally distributed, the combined asset-specific risk would be between 22% and 32% of the magnitude of asset-specific risk for a single property portfolio.

Similarly, the magnitude of combined asset-specific risk for an open-end fund of 100 properties is 25% of the magnitude of asset-specific risk for a single property portfolio. However, if the asset-specific risk were Normally distributed, the combined asset-specific risk would be just 10% of the magnitude of asset-specific risk for a single property portfolio.

Alternatively, if the question of risk reduction is rephrased to ask the number of assets n_k needed in a portfolio to achieve a reduction of asset-specific risk by a specified factor of *k*, then the answer is: n_k is the smallest integer at least as large as *k* raised to the power 1/0.303. In mathematical notation,

$$n_k = k^{\alpha/(\alpha-1)} + 1 \approx k^{3.30} + 1 \qquad (3)$$

This implies that the number of properties in a portfolio needed to achieve a four-fold reduction in the magnitude of combined asset-specific risk is 98—compared with only 16 properties if asset-specific risk were Normally distributed. Similarly, the number of properties in a portfolio needed to achieve a ten-fold reduction in combined asset-specific risk is 1,996—compared with 100 properties if asset-specific risk were Normally distributed. In other words, equally weighted investments in nearly half of the properties currently in the NCREIF database would be needed to achieve a ten-fold reduction in the magnitude of combined asset-specific risk. Quite obviously, no institutional investor has enough assets available for real estate to approach this amount of allocation.

Conclusions

The analysis in this study supports the conclusion that individual (continuously compounded) annual property returns in the NCREIF database are not Normally distributed for calendar years 1980 to 2003, with only two annual exceptions each for Office and Retail properties.

It also supports the conclusion that, for each calendar year t in that interval, there is a stable infinite-variance distribution with Characteristic Exponent α_t such that the annual return on each property can be represented as the average return for that year on properties of the same type plus a random sample from the stable distribution for that year, and furthermore that these samples are independent for distinct properties or calendar years. These stable distributions can be considered to represent real estate asset-specific risk.

The data analysis strongly implies that both the skewness and magnitude of real estate asset-specific risk change over time, i.e., real estate risk is heteroskedastic with respect to both the amount of risk and the shape of the risk distribution.

However, the analysis also supports the conclusion that there is not a single value for the Characteristic Exponent of asset-specific risk across both calendar year and property type. For the aggregate of NCREIF Office, Retail, and Industrial property returns, however, a statistical estimate for the Characteristic Exponent α together with a 95% confidence interval around this value is 1.434 ± 0.022, based on a sample distribution of 33,745 annual property returns over the twenty-four-year sample period. This interval is so far removed from 2.0—the value for a Normal distribution—that it has profound implications for real estate portfolio management and negates risk models built on the presumption of Normality.

The low observed value for the Characteristic Exponent implies that reduction of asset-specific investment risk to levels readily achievable in the stock market through asset diversification requires a portfolio of far more real estate assets than would be needed for the case of Normally distributed risk. In institutional-grade real estate portfolios, the appropriate degree of risk reduction, say 90%, across multiple risk factors (location, economic, etc.) could only be achieved by purchasing most of the institutional-grade properties in the U.S.—an obvious practical impossibility. This implies that institutional real estate portfolio management must be concerned with the asset-specific risk component of each property included in the portfolio with perhaps lesser consideration given to market/systematic and market-sector risk components. In other words, individual asset or property management may be more important for successful commercial real estate investment than portfolio assembly. Furthermore, the stationarity of the Characteristic Exponent for investment risk across time within property type is independent of whether or not regional groupings, for instance, provide a meaningful additional risk dimension as some researchers have suggested.

The fact that real estate investment risk has infinite variance, i.e., the return distributions are statistically distinct from the Normal distribution that is the only stable distribution with finite variance or standard deviation, also implies that there is no way to measure codependence among property risk functions with the statistical tools currently available. Sample correlations used in multi-factor MPT real estate risk models do not measure true risk codependence. The implication or embedded presumption of these multi-factor MPT models that a finite second-moment exists is soundly refuted by empirical tests of the NCREIF data.

A final observation concerns the accuracy of appraisal-based returns data relative to transaction-based data. The fact that thousands of appraisals (or valuations) by real estate professionals across the country over a twenty-four-year period form sample distributions with nearly indistinguishable Characteristic Exponents across calendar years by property types suggests strongly that the real estate community has a common perception of asset value and the sources of that value that have remained constant across changing market regimes of liquidity, tax benefits, credit access, and supply and demand of product. Nonetheless, the existence of outliers for years in which the commercial real estate markets came closest to total gridlock—1991 and 1992—also indicates that real estate professionals require some actual transactions to provide a benchmark for their common estimates of value.

Jumps or discontinuities in reported returns may account for the existence of more high and low values than a Normally distributed set would indicate. Certainly, abrupt changes in rent or occupancy, for example, can and do occur. As sizeable as these unanticipated jumps can be, it is unlikely that these jumps persist for more than a year or two for any single asset. For example, Graff and Young (1999) examined 747 paired appraisals conducted simultaneously by an investment manager and third-party appraisers and found that occasionally the two parties have substantial disagreements about the value of an asset occasioned by extreme vacancy. That one or the other party was proven correct in a subsequent period tended to reduce persistence of high or low returns.

Likewise, the clustering of returns about the mean return for a particular year has a simple explanation. The lease-by-lease discounted cash flow (DCF) technique that has been virtually universal among buyers, sellers, and appraisers of commercial property since the early 1980s creates a conformity or consensus around some particular return that is well known among all industry participants.[7] Interestingly, the consensus estimates are relatively short lived. One calendar year is quite enough time for industry participants to come to some other opinion of the appropriate levels of returns that will clear the market.

The way these new opinions are translated into new estimates of value appear to jump about more than one would expect by the slow pace of changes in market opinion. Once again there is a simple explanation and the same suspect agent, namely the DCF valuation methodology. In short, the DCF is a non-linear function in which small changes in expectations produce large changes in valuation based on the new evidence. It does not take much to create jumps. Variables like discount rates, growth rates, market rent assumptions, and occupancy rates can all produce substantial changes in valuation from the DCF model.

7 In the last decade or more, the conformity has become even more pronounced in the U.S. where almost all market participants use exactly the same analytical software, ARGUS.

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Exhibit 1a
Distribution of Log Annual Total Return Residuals
NCREIF, All Properties, 1980 to 2003

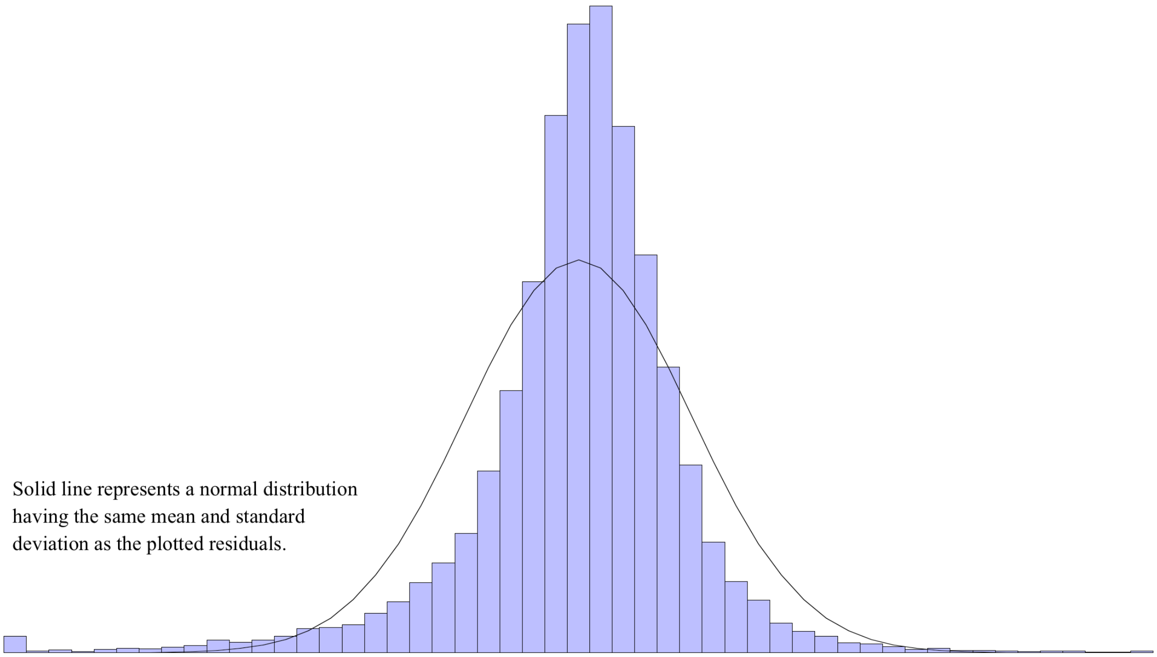


Exhibit 1b
Difference in Frequency, Log Annual Total Return Residuals to Normal Distribution
NCREIF, All Properties, 1980 to 2003

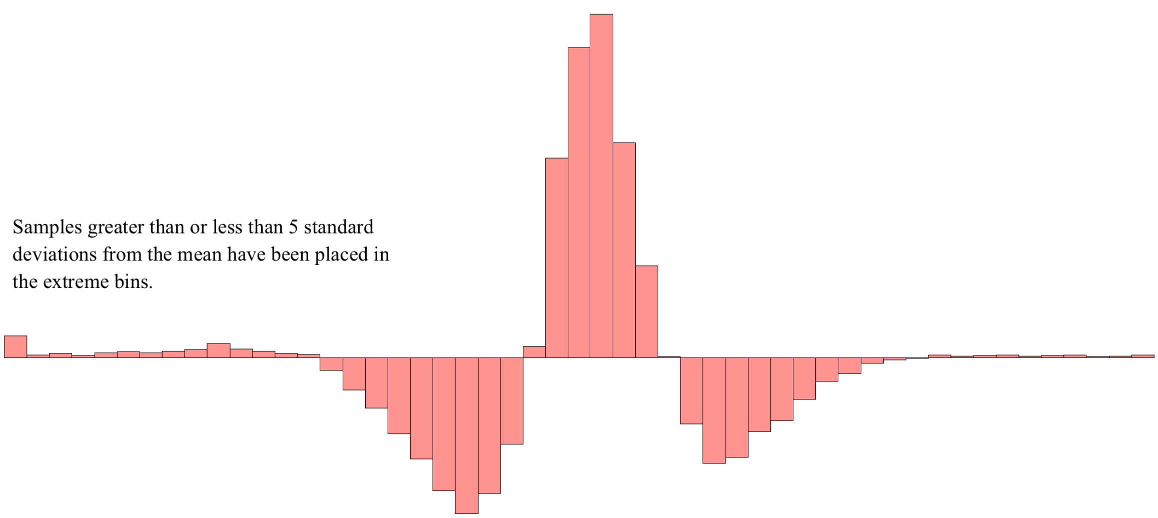


Exhibit 2
Stable Distribution Parameters for NCREIF Property Database
Log Annual Total Return Residuals & Mean Returns & Number of Properties

All Properties Combined:

Year or Period	α	β	γ	Mean Return	Number of Properties
2003	1.428 **	-0.292 **	0.054	0.072	2,644
2002	1.422 **	-0.660 **	0.050	0.051	2,383
2001	1.297 **	-0.364 **	0.041	0.066	2,076
2000	1.269 **	0.231 **	0.039	0.112	1,802
1999	1.325 **	0.076	0.037	0.109	1,632
1998	1.466 **	0.513 **	0.051	0.148	1,549
1997	1.432 **	0.361 **	0.055	0.143	1,548
1996	1.361 **	-0.007	0.048	0.104	1,655
1995	1.372 **	-0.298 **	0.050	0.084	1,454
1994	1.263 **	-0.349 **	0.052	0.064	1,455
1993	1.485 **	-1.000 **	0.069	0.002	1,601
1992	1.594 **	-1.000 **	0.088	-0.051	1,697
1991	1.616 **	-1.000 **	0.088	-0.066	1,641
1990	1.351 **	-0.828 **	0.060	-0.012	1,451
1989	1.332 **	-0.529 **	0.060	0.035	1,299
1988	1.449 **	-0.491 **	0.063	0.052	1,246
1987	1.419 **	-0.586 **	0.072	0.038	1,158
1986	1.452 **	-0.480 **	0.057	0.061	1,099
1985	1.452 **	-0.233 **	0.051	0.100	971
1984	1.325 **	0.060	0.047	0.117	903
1983	1.327 **	-0.158 *	0.053	0.103	889
1982	1.358 **	-0.045	0.051	0.087	692
1981	1.318 **	0.672 **	0.060	0.160	507
1980	1.486 **	1.000 **	0.054	0.155	393
1980-03	1.434 **	-0.292 **	0.059	0.065	33,745
std. dev.	0.011	0.018	0.000		

Exhibit 2 (continued)
Stable Distribution Parameters for NCREIF Property Database
Log Annual Total Return Residuals & Mean Returns & Number of Properties

Office Properties:

Year or Period	α	β	γ	Mean Return	Number of Properties
2003	1.412 **	-0.728 **	0.059	0.035	1,006
2002	1.509 **	-1.000 **	0.063	0.012	928
2001	1.443 **	-0.550 **	0.052	0.047	835
2000	1.265 **	0.316 **	0.043	0.119	682
1999	1.495 **	0.207 *	0.043	0.113	604
1998	1.717 **	1.000 *	0.066	0.174	522
1997	1.754 *	1.000 *	0.075	0.183	430
1996	1.595 **	0.276 *	0.063	0.129	458
1995	1.367 **	-0.178 *	0.066	0.071	394
1994	1.422 **	-0.434 **	0.081	0.049	427
1993	1.468 **	-1.000 **	0.087	-0.031	481
1992	1.551 **	-1.000 **	0.103	-0.115	454
1991	2.000	-1.000 **	0.138	-0.153	457
1990	1.459 **	-1.000 **	0.084	-0.076	420
1989	1.479 **	-1.000 **	0.081	-0.022	399
1988	1.544 **	-1.000 **	0.082	-0.002	376
1987	1.304 **	-1.000 **	0.081	-0.026	358
1986	1.413 **	-0.888 **	0.066	0.019	345
1985	1.471 **	-0.302 *	0.059	0.070	294
1984	1.334 **	-0.082	0.050	0.101	252
1983	1.288 **	-0.283 *	0.053	0.100	237
1982	1.683 *	0.943 *	0.059	0.102	174
1981	1.520 *	0.722 *	0.063	0.173	92
1980	2.000	1.000	0.063	0.153	65
1980-03	1.487 **	-0.405 **	0.072	0.041	10,690
std. dev.	0.021	0.031	0.003		

Exhibit 2 (continued)
Stable Distribution Parameters for NCREIF Property Database
Log Annual Total Return Residuals & Mean Returns & Number of Properties

Retail Properties:

Year or Period	α	β	γ	Mean Return	Number of Properties
2003	1.363 **	-0.111	0.044	0.135	448
2002	1.262 **	-0.090	0.034	0.106	454
2001	1.162 **	-0.437 **	0.030	0.072	458
2000	1.258 **	-0.098	0.033	0.086	454
1999	1.346 **	0.064	0.035	0.107	425
1998	1.248 **	-0.180 *	0.038	0.118	432
1997	1.167 **	0.028	0.037	0.096	470
1996	1.113 **	-0.348 **	0.039	0.059	528
1995	1.317 **	-0.720 **	0.045	0.041	390
1994	1.065 **	-0.402 **	0.035	0.059	379
1993	1.335 **	-0.618 **	0.051	0.034	417
1992	2.000	-1.000 *	0.077	-0.006	386
1991	1.690 *	-1.000 *	0.070	-0.022	380
1990	1.341 **	-0.613 **	0.035	0.049	283
1989	1.120 **	-0.227 *	0.035	0.073	221
1988	1.551 **	0.041	0.057	0.105	206
1987	1.361 **	-0.185	0.048	0.099	196
1986	1.434 **	0.225	0.042	0.111	187
1985	1.268 **	0.012	0.037	0.117	172
1984	1.771	1.000	0.044	0.130	166
1983	1.374 **	-0.077	0.044	0.115	162
1982	1.247 **	-0.112	0.044	0.081	119
1981	1.687 *	-0.768	0.055	0.090	98
1980	1.608 *	0.627	0.044	0.128	74
1980-03	1.337 **	-0.293 **	0.044	0.077	7,505
std. dev.	0.022	0.033	0.001		

Exhibit 2 (continued)
Stable Distribution Parameters for NCREIF Property Database
Log Annual Total Return Residuals & Mean Returns & Number of Properties

Industrial Properties:

Year or Period	α		β		γ	Mean Return	Number of Properties
2003	1.387	**	-0.359	**	0.050	0.081	1,190
2002	1.339	**	-0.634	**	0.042	0.064	1,001
2001	1.257	**	-0.258	**	0.035	0.082	783
2000	1.209	**	0.317	**	0.034	0.124	666
1999	1.132	**	0.060		0.030	0.108	603
1998	1.456	**	0.571	**	0.045	0.146	595
1997	1.506	**	0.834	**	0.053	0.150	648
1996	1.349	**	0.415	**	0.040	0.122	669
1995	1.410	**	0.001		0.043	0.116	670
1994	1.414	**	-0.279	**	0.049	0.077	649
1993	1.784	*	-1.000	*	0.079	0.006	703
1992	1.807	*	-1.000	**	0.096	-0.037	857
1991	1.787	**	-1.000	**	0.088	-0.038	804
1990	1.431	**	-1.000	**	0.060	0.002	748
1989	1.333	**	-0.422	**	0.053	0.056	679
1988	1.377	**	-0.360	**	0.058	0.067	664
1987	1.476	**	-0.596	**	0.070	0.056	604
1986	1.411	**	-0.374	**	0.052	0.071	567
1985	1.533	**	-0.143		0.049	0.112	505
1984	1.252	**	0.083		0.045	0.120	485
1982	1.285	**	-0.162	*	0.049	0.082	399
1981	1.095	**	0.695	**	0.053	0.178	317
1980	1.414	**	1.000	**	0.053	0.164	254
1980-03	1.466	**	-0.279	**	0.057	0.075	15,550
std. dev.	0.017		0.025		0.001		

Statistically significant confidence of non-Normality α ≠ 2.0) or skewness (β ≠ 0):
** = 99% confidence
* = 95% confidence

α is the Characteristic Exponent, and only equals 2.0 for the Normal distribution
β is the Skewness Parameter in the range -1.0 to +1.0
γ is the (positive) Scale Parameter which measures the spread of the distribution about δ

Note: The means are shown in Exhibit 2 for purposes of completeness, but will not be needed for discussion or analysis in the body of this article.

Exhibit 3
Comparison of the Characteristic Exponent for NCREIF Property Database
Between Prior 1995 Study and the Current Study

All Properties Combined:

Year or Period	Prior α	Current α	Current - Prior
1992	1.526	1.594	0.068
1991	1.631	1.616	-0.015
1990	1.348	1.351	0.003
1989	1.329	1.332	0.003
1988	1.489	1.449	-0.040
1987	1.405	1.419	0.014
1986	1.462	1.452	-0.010
1985	1.425	1.452	0.027
1984	1.374	1.325	-0.049
1983	1.376	1.327	-0.049
1982	1.371	1.358	-0.013
1981	1.233	1.318	0.085
1980	1.472	1.486	0.014
averages 1980-92	1.419	1.421	0.003

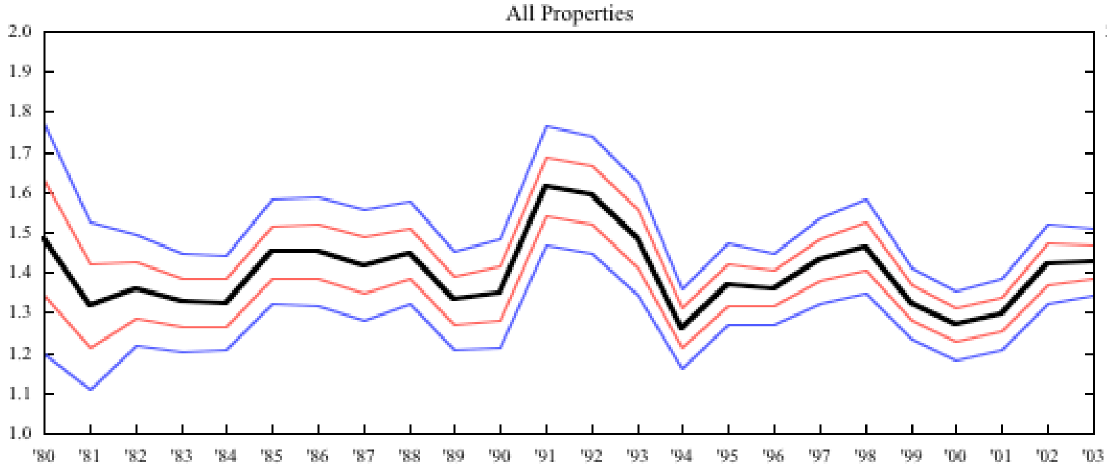
Exhibit 4

Characteristic Exponent "Alpha" of Distributions of Log Annual Total Return Residuals

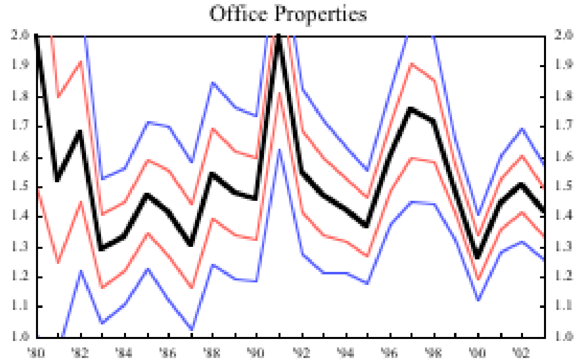
NCREIF 1980 to 2003

(bands indicate plus and minus one and two standard deviations)

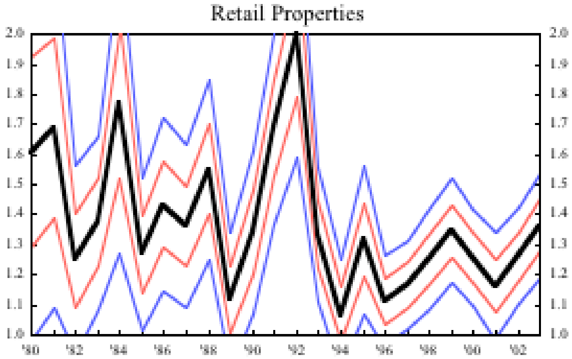
All Properties



Office Properties



Retail Properties



Industrial Properties

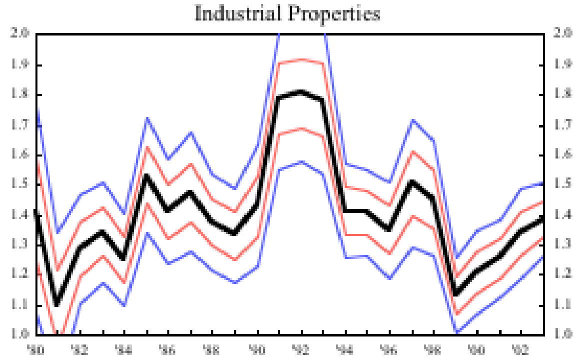


Exhibit 5

Skewness Parameter "Beta" of Distributions of Log Annual Total Return Residuals

NCREIF 1980 to 2003

(bands indicate plus and minus one and two standard deviations)

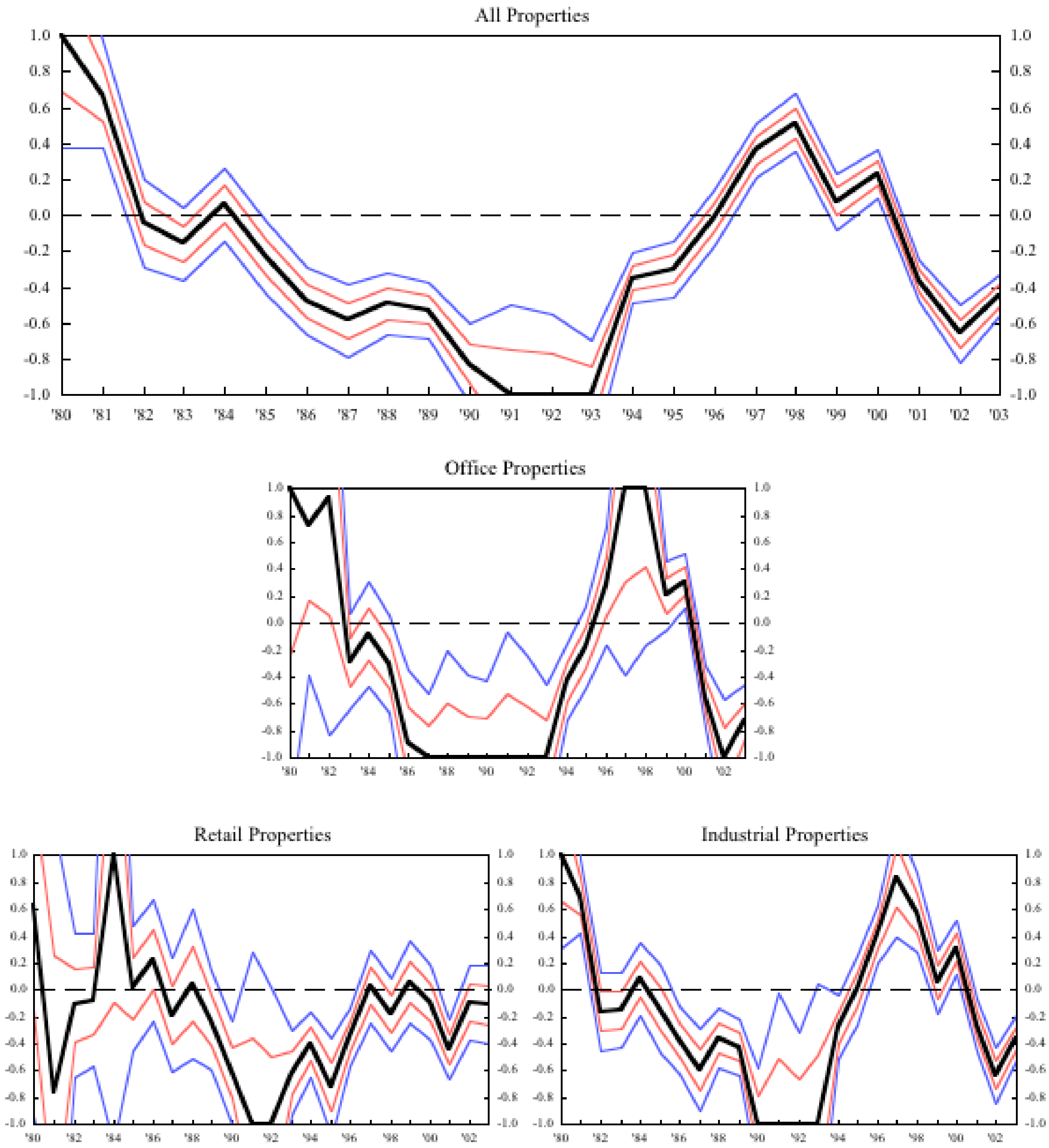
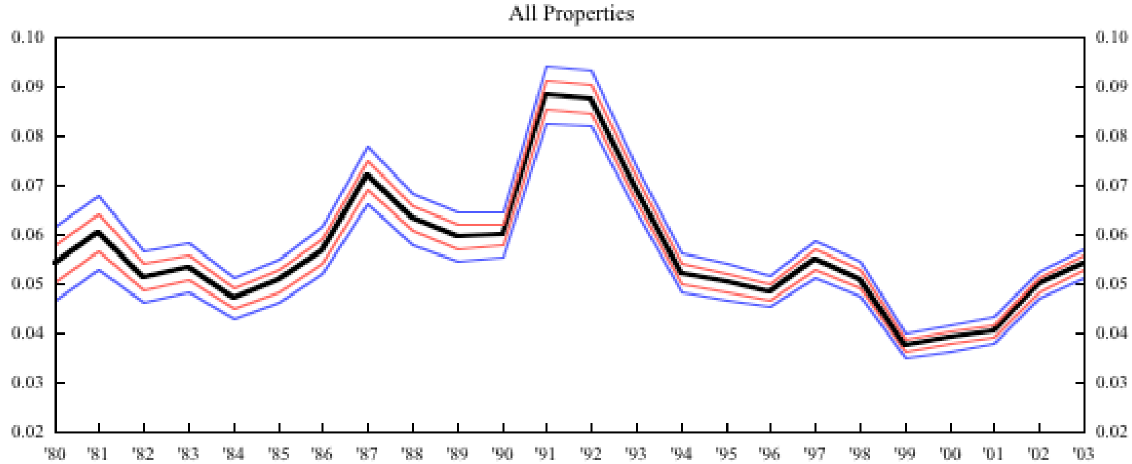
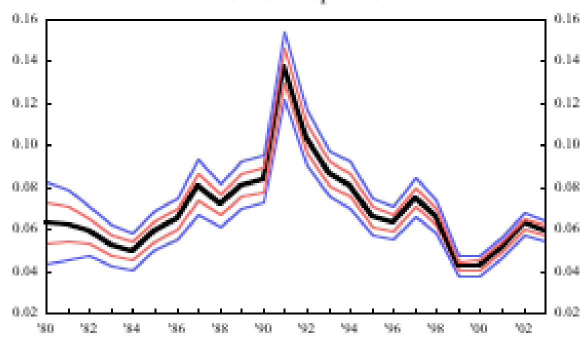


Exhibit 6
Scale Parameter "Gamma" of Distributions of Log Annual Total Return Residuals
NCREIF 1980 to 2003
(bands indicate plus and minus one and two standard deviations)

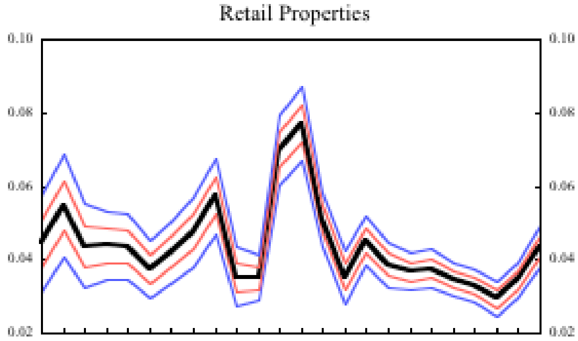
All Properties



Office Properties



Retail Properties



Industrial Properties

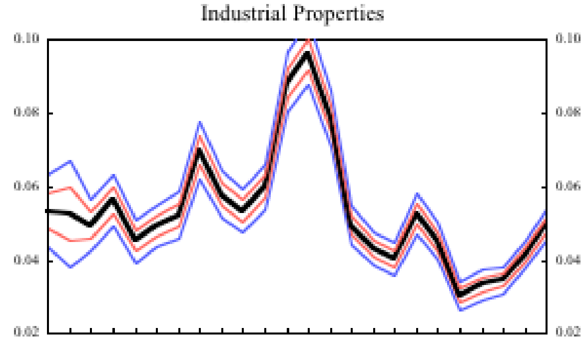


Exhibit 7
Three Parameters of Distributions of Log Annual Total Return Residuals by Property Type
NCREIF 1980 to 2003
(bands indicate plus and minus one and two standard deviations)

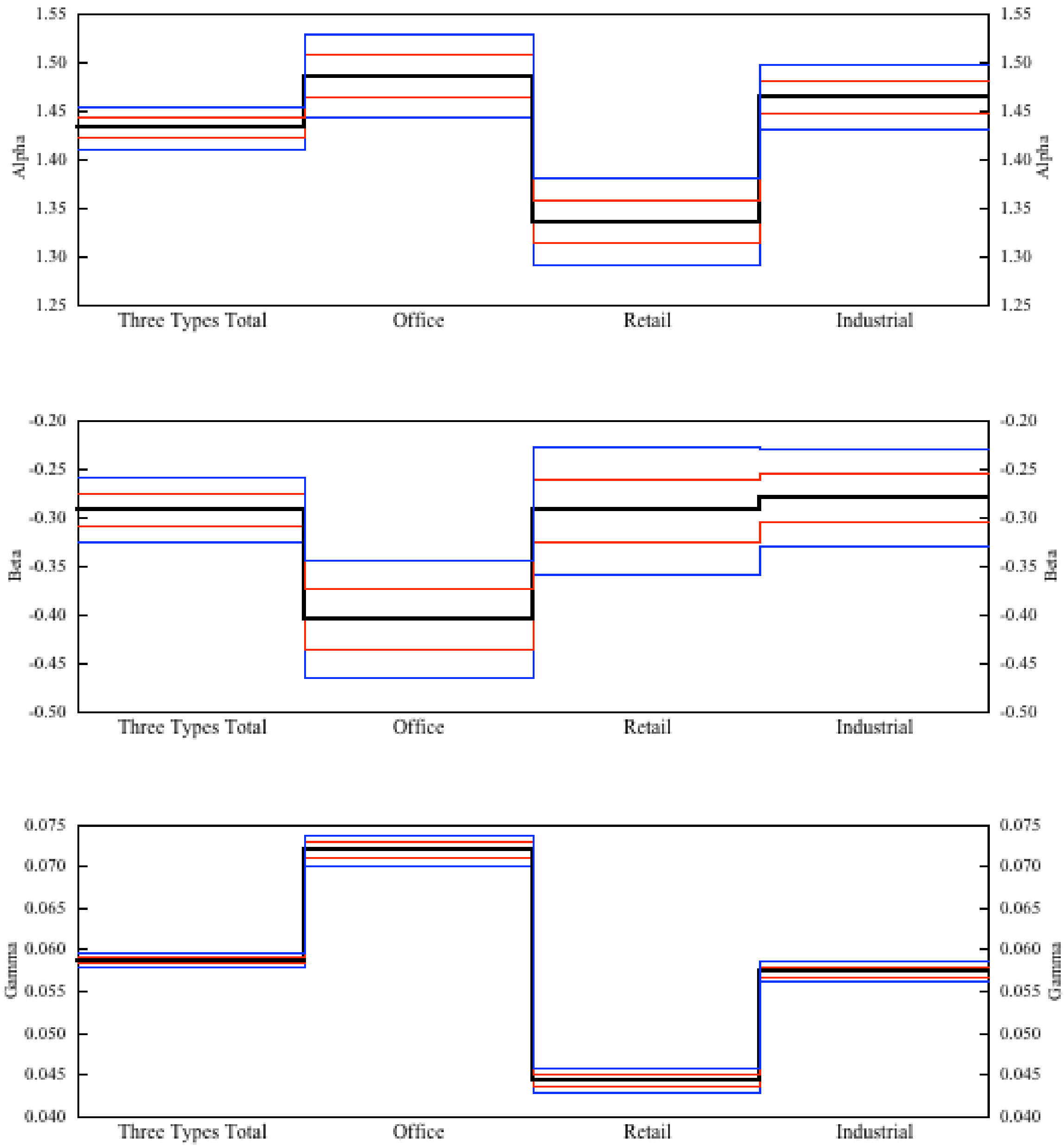


Exhibit 8
Comparison of Stable Distribution Parameters in U.S., U.K., and Australian Studies
(standard errors in parentheses)

	U.S., NCREIF	U.K., IPD	Australia, PCA
Characteristic Exponent, α			
All Properties	1.434 **	1.448 **	1.588 **
	(0.011)	(0.004)	(0.035)
Office	1.487 **	1.431 **	1.649 **
	(0.021)	(0.007)	(0.050)
Retail	1.337 **	1.471 **	1.554 **
	(0.022)	(0.006)	(0.072)
Industrial	1.466 **	1.425 **	1.635 **
	(0.017)	(0.009)	(0.074)
Skewness, β			
All Properties	-0.292 **	0.136 **	-0.242 **
	(0.018)	(0.006)	
Office	-0.405 **	0.053 **	-0.279 *
	(0.031)	(0.012)	
Retail	-0.293 **	0.257 **	-0.186
	(0.033)	(0.008)	
Industrial	-0.279 **	-0.025 *	0.037
	(0.025)	(0.015)	
Scale Parameter, γ			
All Properties	0.059	0.066	0.089
	(0.000)	(0.000)	
Office	0.072	0.072	0.099
	(0.003)	(0.000)	
Retail	0.044	0.066	0.060
	(0.001)	(0.000)	
Industrial	0.057	0.056	0.069
	(0.01)	(0.000)	
Sample Size, Number of Properties			
All Properties	33,745	269,853	4,593
Office	10,690	81,121	2,591
Retail	7,505	138,993	1,023
Industrial	15,550	49,739	979

Statistically significant confidence of non-Normality $\alpha \neq 2.0$) or skewness ($\beta \neq 0$):
** = 99% confidence
* = 95% confidence

U.S. data from NCREIF, 1980 to 2003
U.K. data from IPD, 1981 to 2003
Australia data from Property Council of Australia, 1985 to 1996