Real Estate Return Distributions with New NCREIF Data Series

by

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Abstract: The accuracy of real estate return distribution parameter estimation is affected by the tools used to do the work as well as the data sets employed. Consistent with previous studies, investment risk models with infinite variance describe distributions of individual property returns in the new NCREIF Indicators: Capital Performance and Property Operations individual property database over the period 1990 to 2014. Applying Maximum Likelihood Estimation (MLE) to historic data shows real estate investment risk to be heteroscedastic, but the Characteristic Exponent of the investment risk function varies more among property types than previously reported whether computed by MLE or other estimation techniques.

Key words: Asset-specific risk, return distributions, Maximum Likelihood Estimation, non-Normality, diversification, institutional investing, NCREIF

Introduction

The technology employed to define the shape of financial asset return distributions has been a persistent, if sporatic, effort for over fifty years. With each new attempt has come some improvement in precision and greater appreciation for just how far removed financial asset return distributions are from Gaussian Normal distributions, the default presumption in countless academic studies, educational courses, and applications.

Shortly after cave painting came double-log graph paper upon which Benoit Mandelbrot (1963) plotted cotton prices over a sixty-year period using a #2 lead pencil. Regardless of the time interval he chose, the patterns always looked the same, namely a cyclical, non-periodic pattern with respect to scaling, but an erratic pattern with respect to a Normal distribution of price levels. Had the return distributions been

Gaussian Normal, the line connecting the points would have been a straight line, but that was not to be. With departures from the straight-line expectation more pronounced at the high positive and low negative ends of the plot, Mandelbrot concluded that the distributions were not likely to be identically, independently distributed (iid) Normal but some other Levy-stable distribution with infinite or undefined variance.[1]

While the fractal view of the world has been shown repeatedly to be a superior representation of the real world, the predominant view espoused in academia and among practioners is that simplified models rooted in easily-taught and easily-comprehended models are "good enough."

Being just "good enough" may suffice for teaching general principles, but it can fall woefully short of providing tools for practical applications of those principles. One case in point is the degree to which adding uncorrelated risky assets to a portfolio might reduce the non-systematic risk of the portfolio. Non-systematic risk reduction at the rate of *one over the square root of the number of assets* is commonly taught in investment classes and espoused by investment managers. However, research by Young and Graff (1995) has shown that a more accurate picture of commercial property return distributions implies non-systematic risk reduction at the *rate of about one over the cube root of the number of assets*.

These differences are non-trivial (as shown graphically in Exhibit 12a and b). Where it would take 100 assets whose returns were independent and Normally distributed (i.e., having a Levy-stable Characteristic Exponent of 2.0) to affect a ten-fold reduction in non-systematic risk in a portfolio, it would take 1,000 assets to produce the same result for assets whose returns approximate the empirical results of studies by Young and Graff (1995) and Young (2008) (i.e., having a Levy-stable Characteristic Exponent of 1.5).

As in the prior work on real estate return distributions in Young and Brown (2012), this article uses the latest Maximum Likelihood Estimation (MLE) techniques, with confidence interval estimation from Nolan, both departures from earlier work in the US, Australia, the UK, and Germany.

However, this is the first research use of NCREIF's new data series formally called "NCREIF Indicators: Capital Performance and Property Operations" that includes two related sets: the Market Value Index (MVI), the Free Cash Flow Yield (FCFY). We sum the individual MVI and the FCFY quarterly return statistics for each property and then chain-link four calendar quarter sums to create annual total returns for commercial property from 1990 to 2014 disaggregated by four property types: Office, Retail, Industrial, and Apartment. Importantly, these series differ from the

NCREIF Property Index (NPI) that has been used in all previous studies on domestic commercial real estate return distributions.

The major difference between the NPI and the MVI+FCFY series is that the NPI includes properties that, from time to time, incur substantial capital expenditures for renovation, expansion, partial sales or repurposing. These expenditures have a significant impact on total returns reported in the NPI both positive and negative.[2] More importantly, the properties incurring these capital expenditures change their physical, functional, or financial economic character as a result of the expenditure. In a real sense, these properties are not of the same character pre- and post- major capital expenditures or, in common real estate parlance, they are not "same store."

The new NCREIF data series address this "same store" problem by applying filter rules to exclude properties and their computed results in quarters when major capital expenditures are incurred. Owing to accounting conventions, major capital expenditures can be both either positive or negative in amount and accordingly total returns in the NPI when these expenditures happen may increase or decrease substantially, often at rates that exceed 100% plus or minus in a single quarter. By removing quarters when this happens, the remaining quarterly total returns in the data set computed as the sum of market value change and distributable net cash flow (MVI+FCFY) seldom push the results beyond logical or realistic bounds and will be somewhat different than total returns in the classic NPI as will become clear later. It also helps confirm that heavy tails, when observed, are not the result of accounting anomalies.

See Young, Fisher, and D'Alessandro (2017) for a more complete description of the NCREIF Indicators: Capital Performance and Property Operations data series, its development, and its uses. Also, in light of the unique filter rules employed and the fact that this article is the first published research utilizing the new data series, a section below will include an extensive descriptive quotation about the filter rule from this *Journal of Real Estate Literature* article.

Levy-stable Distributions in Real Estate

Starting in the early 1970s, the degree to which Modern Portfolio Theory (MPT) and Efficient Markets Hypothesis (EMH) came to dominate the investment world was as remarkable as it was contrary to traditional thinking about how theories may be derived from empirical evidence. After nearly two decades of influence in the stock market, MPT and EMH were introduced to real estate without empirical justification. In Young and Graff (1996) the authors assert: "MPT and EMH seem to have been introduced into real estate to justify the use of particular statistical techniques and portfolio strategies rather than as a consequence of empirical analysis of investment return and risk characteristics. In science, the situation is generally reversed: theories are developed to explain observations."

In a departure from the facile presumption of Gaussian normal distributions, McCulloch's quantile-based methodology (McCulloch (1986)) has proven robust for the analysis of real estate return distributions in the United States (Young and Graff (1995), Brown (2000), and Young (2008)), Australia (Graff, Harrington, and Young (1997)), the United Kingdom (Young, Lee, and Devaney (2006)), and, more recently, in Germany (Richter, Thomas, and Fuss (2011)).[3]

MLE has surpassed McCulloch's estimators in ease of use for both academics and practitioners alike because fast, inexpensive computing is now ubiquitous. MLE applications have been available in the Fortran programming language for two decades, ten years ago these routines were added to the kernel of Wolfram Research's symbolic computing software Mathematica.

The idea behind MLE of distribution parameters is to derive the various parameters which maximize the probability that the parameters best describe the sample distribution. While seemingly simple in concept, MLE is numerically intense, which makes the availability of powerful computers for data processing necessary.

For those inclined toward empirical analysis of sample data, the advent of powerful software coupled with the availability of fine-grained performance data offer an opportunity to (1) examine the real world characteristics of real estate as an investable asset class, (2) probe for similarities and differences among discrete physical and functional dimensions like property types or location, (3) develop strategies and tactics to take advantage of persistent similarities and differences, (4) test alternative measures of real estate risk that could be used to mitigate potential losses or enhance portfolio performance, and (5) finally abandon the mathematics of MPT altogether as a technical construct of finance that is unworkable in real estate.

For our purposes, perhaps the most useful byproduct of MLE's good statistical properties is its ability to provide confidence intervals to quantify uncertainty within the data. With respect to McColluch's estimates of confidence, modern MLE measures are a substantial improvement in precision. In the results that follow, for example, the magnitude of the standard errors for the characteristic exponent from MLE are between one-quarter and one-third the size of those produced by McCulloch's quantile model.

By application of MLE as employed in Young and Brown (2012), this article tests whether more accurately stated property return distributions have finite variance and are Gaussian Normal. We also compare those results to the prior article for the overlapping period 1990 to 2010. The short answer is that they still are not Normal over the four major property types. The longer answer confirms the same for the combined annual data series with equal precision and statistical confidence despite the differences in the number and individual property returns of properties within the new NCREIF Indicators: Capital Performance and Property Operations data compared to the classic NCREIF Property Index (NPI).

Levy-stable Distributions

Levy-stable distributions offer a number of useful properties. First, they are shape preserving under addition and linear transformation, comporting with a long held premise of the finance paradigm that prices are the result of an accumulation of randomly arriving information. This permits a host of useful results in mathematical statistical theory which relate to sums of random variables. Second, they appeal to the Generalized Central Limit Theorem which holds that if a distribution has a limiting distribution, it must be a member of the Levy-stable class. Normal distributions are Levy-stable and, importantly, are the only Levy-stable distributions with finite variance.

The log characteristic functions of Levy-stable distributions have the following form for cases where $\alpha \neq 1$:

$$\psi(t) = i\delta t - |\gamma t|^{\alpha}[1 - \beta sgn(t)\tan(\pi\alpha/2)] \qquad (1)$$

The four parameters α, β, γ, and δ in Equation (1) completely characterize the distribution.

The Characteristic Exponent α lies in the half-open interval (0,2] and measures the rate at which the tails of the density function decline to zero. The larger the value of the Characteristic Exponent α, the faster the tails shrink toward zero. When α=2.0, the distribution is Normal.

While the means (first moments) of Levy-stable distributions with Characteristic Exponents α>1.0 do exist, variances (second moments) do not exist—i.e., are infinite––for those distributions with Characteristic Exponents α<2.0.

The Skewness Parameter β lies in the closed interval [-1,1], and is a measure of the asymmetry of the distribution. The closer the Characteristic Exponent α is to the upper limit of the permissible range, the less significance the skewness has in terms of shifting the shape of the distribution away from the corresponding symmetric distribution. At the limit α=2.0, the Normal distribution, the Skewness Parameter β becomes irrelevant and all Levy-stable distributions are symmetric.

The Scale Parameter γ lies in the open interval $(0,\infty)$, and is a measure of the spread of the distribution. If α=2.0, the Scale Parameter γ is directly proportional to the standard deviation: $\gamma=\sigma/\sqrt{2}$. However, the Scale Parameter γ is finite for all Levy-stable distributions, despite the fact that the standard deviation is infinite for all $\alpha<2.0$. Thus, the Scale Parameter γ can be regarded as a generalization of the standard deviation, first reliably quantified by McCulloch (1986).

The Location Parameter δ is a rough measure of the midpoint of the distribution. A change in δ simply shifts the graph of the distribution left or right, hence the term "location."

There are a number of parameterizations of stable laws (Nolan (1997, 1998, and 2005)). Two are predominant in financial applications. Nolan's S0 is useful in theoretical work as it is continuous in all four parameters; Nolan's S1 is often used because the location parameter is the mean.

NCREIF Indicators: Capital Performance and Property Operations

Since its inception in 1978, the NCREIF Property Index (NPI) has been a measure of investment performance, a measure of returns in total and returns decomposed into income and capital subsets and disaggregated by various property characteristics and geographic locations. In a sense, the NPI is a measure of returns on a "portfolio" of institutionally-owned commercial real estate, which is why the headline returns reported are value-weighted. Fortuitously, NCREIF has collected more information about the economic and operating performance of domestic commercial property since 2000 that can be extracted and reconstructed to create other data series.

Furthermore, unlike most other real estate data series, NCREIF financial data are audited, accounting-based data because the source is information provided by owners or managers who are themselves fiduciaries subject to the hightest standard of care under the law (29 U.S. Code § 1101-1114).

In 2015, NCREIF released three refined data series called the Market Value Index (MVI), the Free Cash Flow Yield (FCFY), and the Capital Expense Ratio (CXR) collectively known by the formal name NCREIF Indicators: Capital Performance and Property Operations. The first two series are used in this paper for the simple reason that the sum of MVI and FCFY in any quarter is the quarterly total return for the property. The percentage amounts can be added to get the total return because both the MVI and the FCFY share the same denominator, namely the market value at the beginning of the quarter. This simple denominator is also an important departure from the more complicated one employed in the classic NPI.

In accounting terms, the MVI is the unrealized gain from an appraisal-based change in value rather than from the actual sale of the asset and the FCFY is the net cash flow that could be distributed to the property owner. Together these statistics represent the capital and net cash income gains on a property in a quarter. At first glance, these terms, capital and income gains, might appear similar to Price and Cash Flow Indices that NCREIF has provided researchers and practitioners in detailed spreadsheets or in on-line query tools.

However, comparing the classic NPI Price and Cash Flow formulas with the MVI and FCFY formulas will demonstrate that these various measures are indeed quite different. It would not be an exaggeration to say that the Price and Cash Flow formulas are, at best, improperly named and often misused owing to the false impression their names denote rather than connote.[4]

The NPI formula is:

$$R_t = \frac{MV_t - MV_{t-1} + PS_t + NOI_t - CI_t}{MV_{t-1} - \left({PS_t}/{2}\right) - \left({NOI_t}/{3}\right) + \left({CI_t}/{2}\right)} \tag{2}$$

where R_t is the Total Return for period t, MV_t is the Market Value at the end of period t, MV_{t-1} is the Market Value at the beginning of period t, PS_t is any Partial Sales in period t, NOI_t is the Net Operating Income in period t, and CI_t is the Capital Expenditures (Improvements) in period t.

The Price (P_t) change for a quarter is:

$$P_t = \frac{MV_t - MV_{t-1} + PS_t}{MV_{t-1} - \left({PS_t}/{2}\right) - \left({NOI_t}/{3}\right) + \left({CI_t}/{2}\right)} \tag{3}$$

The Cash Flow (CF_t) for a quarter is:

$$CF_t = \frac{NOI_t - CI_t}{MV_{t-1} - \left({PS_t}/{2}\right) - \left({NOI_t}/{3}\right) + \left({CI_t}/{2}\right)} \qquad (4)$$

Significantly, within the classic NPI computational formula, there are two issues related to the calculation of the so-called Price and Cash Flow Indices. First, the same denominator is used as in the NPI. This denominator is based on the Modified Dietz Method (Dietz, 1966) and designed to provide an estimate of an Internal Rate of Return (IRR) for a *single quarter assuming monthly cash flows*. However, as constructed, the Price Index and Cash Flow Index *were not intended to be returns*. Second, the ending value of the Price Index will be higher by all capital expenditures including capital spent for expansion and major renovations not just ordinary, routine capital expenses generally associated with tenant leases such as commissions and modifications to or construction of new tenant spaces. Notably, a property undergoing expansion or major renovation is not the same property at the end of its accounting period as it was before the renovation or expansion. In other words, the Price Index is not a "same store" analysis of price changes; the store has changed substantially physically, functionally, and economically in the quarter as a result of the owner's strategic or business decision.

The Market Value Index (MVI) is designed to deal with these issues in two ways. First, the denominator of the MVI formula is simply the beginning market value for the quarter: MV_{t-1}. This treatment is more consistent with the way price indices are calculated for other asset classes. Second, the MVI excludes properties in quarters where there is major renovation or expansion. These exclusions eliminate computations in quarters where a property is likely changing its physical, functional, or financial economic character.

Thus, the MVI is an equally weighted average of quarterly changes in reported market value pure and simple with no major capital expenditures involved. It is a refinement of the NCREIF Query Tool's "Price Index" in that it is generated strictly from period-to-period changes in the reported market value for each property rather than from changes in the Capital Return including Capital Expenditures, which is a return measure inaccurately described as a change in price.

The Free Cash Flow Yield (FCFY) is a measure of the amount of a property's quarterly operating net cash flow available for distribution to investors/owners expressed as a fraction of market value. This is a *periodic measure that is not indexed*, that, like the MVI, excludes properties with major capital expenditures for expansion or renovation when such expenditures exceed the filter rule described below, and that more accurately reflects the actual net cash that could be distributed to investors or to property owners in a quarter. This measure is similar to the concept of "free cash flow" used in the finance literature and in corporate financial reporting.

The FCFY is a periodic measure of the cash that investors can expect from operation of commercial property. In some sense, it is the cash in hand each quarter after all operating and everyday capital expenses have been paid. To many investors, the cash available for distribution is one of the principal reasons for investing in commercial real estate, an asset that has a relatively stable net cash flow stream due to the terms of leases that underpin a property's economic performance. Stability and reliability of this net cash flow stream is important to a wide range of institutional investors who must have cash available for distribution to plan beneficiaries. With the introduction of the FCFY, those investors have, for the first time, a way of assessing the history of net cash flow available for distribution from the real estate asset class.

In summary, the new measures introduced in the NCREIF Indicators: Capital Performance and Property Operations quarterly data sets differ from the NPI in several ways:

1. Because the new measures that sum to the quarterly total return, namely MVI and FCFY, are not intended as estimates of a quarterly IRR, no adjustment has to be made to the denominator of the NPI's formula for estimated cash flows within the quarter. The denominator is simply the Beginning Market Value.
2. Unlike the NPI Income Return component based on NOI, the FCFY is calculated by subtracting ordinary, predominantly lease-related Capital Expenditures, from the NOI to get a measure of net cash flow.
3. The MVI, unlike the NPI Appreciation Return component where all Capital Expenditures are deducted from the Ending Market Value, there is no deduction for Capital Expenditures. Thus, the MVI simply measures the change in Market Value plus Partial Sales, if applicable.
4. While some Capital Expenditures are necessary to maintain a constant utility of a property, a price or value index should not reflect an increase in the value of the property due to a major expansion or renovation that involves new capital investment. Similarly, the FCFY measure should not have major, extraordinary Capital Expenditures deducted. Thus, for both the MVI and the FCFY, a property is excluded from the calculation during any quarter where expansion or major renovations are taking place.
5. The NPI returns have always been value-weighted as they represent the universe of properties reported to NCREIF. In a sense, the NPI returns are the returns of a particular "portfolio" of institutionally-owned commercial real estate. The new measures are equal-weighted to reflect data representative of the universe of domestic commercial real estate

regardless of ownership. When dealing with sample statistics from a universe of domestic commercial real estate, equal-weighted statistics are preferable.

MVI and FCFY Defined

MVI is computed for each qualified property as the sum of Ending Market Value and Partial Sales[5] divided by Beginning Market Value minus 1.0 for each quarter. To deal with the “same store” issue discussed above, in any quarter where the absolute value[6] of specified capital expenditures exceeds a fraction of Beginning Market Value the property’s MVI computation is disqualified or excluded from the data series.

The MVI formula is:

$$\left[{(MV_t + PS_t)}/{MV_{t-1}}\right]-1 \qquad (5)$$

or alternatively

$$(MV_t - MV_{t-1} + PS_t)/{MV_{t-1}} \qquad (6)$$

where MV_t is Market Value and PS_t is Partial Sales reported to NCREIF in quarter t and MV_{t-1} is the Market Value at the end of the prior quarter, in other words, at the beginning of the current quarter.

FCFY is computed for each property as the quantity Net Operating Income minus Capital Improvements divided by Beginning Market Value for each quarter. Notice that major capital expenditures for expansions or renovations are not included in the formula, while ordinary, frequently occurring capital expenses related to leasing commissions, tenant improvements, and other routine building maintenance are included in the term CI_t. Accordingly, in quarters where the absolute value of capital improvements defined in the filter rule exceed a fraction of Beginning Market Value, the property’s FCFY computation is excluded from the data series.

Thus, the FCFY formula is:

$$(NOI_t - CI_t)/{MV_{t-1}} \qquad (7)$$

where NOI_t is the Net Operating Income, CI_t is Capital Improvements related to tenant expenses or retention and routine capital expenditures, and MV_{t-1} is Market Value reported to NCREIF at the end of the prior quarter.

As mentioned above, MVI_t and $FCFY_t$ are components of total return for a quarter when substantial capital expenditures have not occurred. Expressed in algebraic form, the total return for a property in a quarter would be simply:

$$TR_t = MVI_t + FCFY_t \tag{8}$$

The Data Series Filter Rule

One of the essential differences between the classic NCREIF NPI returns series and the new NCREIF Indicators: Capital Performance and Property Operations data series is the filter rule that excludes particular NPI properties. How this filter rule was crafted is so consequential to an understanding of the new data series that we believe that an extended quotation from Young, Fisher, and D'Alessandro (2017) is warranted.

> To ensure that, within reasonable bounds, a particular property retains its physical continuity throughout a quarter, there must be a way to identify properties that have not had substantial, material changes to the physical asset within the quarter. If the changes are substantial, the property should be excluded for that quarter or for subsequent quarters until such time as the property becomes stable physically, functionally, or economically, i.e., when the property returns to a state of constant utility.
>
> Prior to 2000, only total Capital Improvements were reported to NCREIF. Subsequently, additional subcategories of capital improvements gave us more information on the composition of total Capital Improvements. In particular, the subcategories included Additional Acquisitions Costs, Leasing Commissions, Tenant Improvements, Building Improvements, Building Expansion, and Other Capital Improvements.
>
> We divide these subcategories into two groups: those that are typical recurring capital expenses related to changing tenancy and ordinary repairs, and those that are occasional, high-dollar-value capital expenditures that alter the physical, functional, or economic condition of a property. Leasing Commissions, Tenant Improvements, and Building Improvements fall into the former group and are included in Capital Expenses in the FCFY … series. Additional Acquisitions Costs, Building Expansion, and Other Capital Improvements fall into the latter group and are all candidates for filtering properties for exclusion within all three series.
>
> We can use the detailed data on capital expenditures from the post-2000 era to create filter rules for excluding properties undergoing substantial capital expenditures prior to 2000. We are not able to say with certainty that properties filtered will be 100%

accurately identified. We must strike a reasonable balance based on indicators we find in the existing data and judgments about the reasonableness of the filter ratio.

Thus, we have chosen to filter only those subcategories of capital expenditures in the subcategories of Additional Acquisitions Costs, Building Expansion, and Other Capital Improvements that show an absolute value greater than 5% of Beginning Market Value of a property in any quarter. First, we compute for each property type the fraction of post-2000 observations that are filtered and then use that fraction to establish a filter rule for the pre-2000 era where we have only one statistic for total Capital Improvements.

We tried several filter rules for pre-2000 data and found that an absolute value of total Capital Improvements greater than 10% of Beginning Market Value provided the most similar fraction of excluded quarters for most property types and for the aggregate of all properties in the NPI. Exhibit 1 [*not included in this quotation*] shows the observations and fractions of properties excluded in both the pre-2000 and post-2000 eras. In the post-2000 era, 2,079 of the 289,543 quarterly observations for all properties in the NPI were filtered (satisfied the rule for exclusion from the data set), a total of 0.72% of all observations. Interestingly, in the pre-2000 era, 958 of the 132,635 quarterly observations for all properties in the NPI were filtered, also a total of 0.72% of all observations despite some differences in percentages pre- and post-2000 when disaggregated by individual property type.

The net effect of this filter rule upon the number of annual observations analyzed in this article relative the prior MLE-based article in the *Journal of Portfolio Management* by the authors covering the same 1990 to 2010 period is shown in Exhibit 1. In this article, there are 1,140 fewer annual observations or approximately 1.9% fewer than in the prior article. The reduced sample sizes by property type over the 1990 to 2010 period are 281 or 1.8% for Office properties, 222 or 2.1% for Retail properties, 467 or 2.1% for Industrial properties, and 170 or 1.3% for apartment properties. As could be surmised from these results, Retail and Industrial properties had a greater relative number of quarters with major capital expenditures than Office properties, perhaps a somewhat unexpected result but consistent with the fact that Office properties within the NCREIF database are generally multi-story buildings on sites with little room to expand. On the other hand, the relatively few observations filtered from the Apartment property type set is consistent with the fact that most capital expenditures in the Apartment sector tend to be frequent, small scale modifications to individual apartment units, common areas, or HVAC systems, rather than property expansion or repurposing.

Analytical Tools for Levy-stable Distributions

A variety of commercially-available computer software products contain statistical routines including Mathematica, MATLAB, Maple, SAS, and SPSS; an open-source, free application known as R; as well as spreedsheet applications like Microsoft's Excel and Apple's Numbers. Statistical routines for the analysis of Levy-stable distributions is less commonplace, however.

One notable exception is Wolfram Research's Mathematica, a symbolic-logic computing software application that offers an extensive suite of statistical tools including Levy-stable distributions. Independent applications written in different source code such as Fortran can be integrated with standard Mathematica tools via a sub-routine know as Mathlink. John Nolan's STABLE for Mathematica, DOS-based Fortran application that uses Mathlink is available at his Stable distribution web site:

http://academic2.american.edu/~jpnolan/stable/stable.html.

The standard error and confidence interval results of this paper were computed with Nolan's STABLE application.

Tests and Results

Before fitting Levy-stable distributions to the sample data, we corrected for possible extraneous data dispersion by reducing each annual return by the corresponding sample mean for that calendar year and property type. The means are shown in Exhibits 3 and 6 for purposes of completeness, but will not be needed in the subsequent discussion.

Because this article uses a NCREIF data set that differs from NCREIF's classic NPI series, the data set underlying all previous studies of real estate return distributions in the US, we begin by comparing the number of observations in the NPI and the MVI+FCFY in the period 1990 to 2010 that is the period studied using MLE in Young and Brown (2012). Exhibit 1 shows that there are 1,140 fewer MVI+FCFY observations than NPI observations for a difference of 1.9%. Similar comparisons by property type also show fewer MVI+FCFY observations: Office properties have 1.8% fewer observations, Retail properties 2.1% fewer, Industrial properties 2.1% fewer, and Apartment properties 1.3% fewer.

Using Mathematica's Maximum Likelihood Estimation (MLE) routine, we fit a Levy-stable distribution to each set of residuals arranged by calendar year and property type over the calendar years 1990 to 2014. To test whether the parameters varied during the sample period, Levy-stable parameters were estimated for sets composed of the residuals aggregated across calendar years and property types. These results are tabulated in Exhibit 6 and are displayed graphically together with 99% confidence intervals (for all years where standard errors could be ascertained) in the Exhibit 5 for the parameter α, β and γ (δ, the Location Parameter, is irrelevant as an estimator of the mean for our purposes because our analysis adjusts for the effect of time-varying means by taking residuals for each calendar year).

Again, we note that there are differences in between the NPI-based series and the MVI+FCFY-based series in the 1990 to 2010 period for the important Characteristic Exponent α. The differences are tabulated for the aggregate and property-type disaggregated data sets in Exhibit 2. On average, the MVI+FCFY Characteristic Exponents are slightly greater than the Caracteristic Exponents for the NPI data sets. For the aggregate data, the average α is 1.547 for the MVI+FCFY versus 1.535 for the NPI, a difference of +0.012. Average differences in α by property type are +0.042 for Office properties, +0.014 for Retail properties, +0.013 for Industrial properties, and +0.012 for Apartment properties.

Despite the small increases in estimated α in the MVI+FCFY series versus α reported in earlier NPI-based studies, the Characteristic Exponents α still evidence statistically significant departures from Normality with 99% confidence. Covering the study period 1990 to 2014, the estimated Characteristic Exponent α for All Properties in the MVI+FCFY series is 1.596. Corresponding estimates by property types are 1.629 for Office properties, 1.531 for Retail properties, 1.618 for Industrial properties, and 1.652 for Apartment properties.

Over the same period, the return distributions are negatively skewed (Skewness Parameter β) for All Properties and all property-type disaggregates except Apartments. This result is directionally identical to the results reported in Young and Brown (2012).

With respect to the Scale Parameter γ, a generalization of the standard deviation or, more commonly, the risk of an asset, the figures tabulated in Exhibit 3 are marginally smaller than the corresponding figures in Young and Brown (2012), albeit the two studies cover overlapping but not coincident time periods. No doubt, the filter rule employed in the construction of the MVI and FCFY series reduced the magnitude of abnormally high and low reported returns thereby producing this outcome. The Scale Parameter γ for All Properties is shown as 0.059 with ranges from a high of 0.071 for Office properties and a low of 0.051 for Apartment properties.

Given the popular anthropomorphic description of leptokurtic distributions relative to Normal distributions, namely leptokurtic distributions have fat tails (okay, that is not really anthropomorphic), weak shoulders, and tall bodies with pointed heads, it may be of interest to note that a reasonably close approximation of the Scale Parameter γ can be derived from the Semi-Interquartile Range easily obtained from ordinary parametric statistics of real estate return distributions. Exhibit 4 shows dispersion estimates of the MVI+FCFY series for Standard Deviation or Second Moment of the Normal Distribution, Semi-Interquartile Range, and the Scale Parameter γ. For All Properties, the Semi-Interquartile range is 0.055 and the Scale Parameter γ 0.059 for a difference of only –0.004. Similarly, small differences are found across all four property-type disaggregations.

In the case of Characteristic Exponents α_t estimated by calendar year and property type, 100% of the samples tabulated in Exhibit 3 (for which standard errors could be computed) by property type were distinct statistically from 2.0, the Characteristic Exponent of the Normal distribution, with 99% confidence. In the case of residuals aggregated across property type, all sample Characteristic Exponents α_t were distinct from 2.0 with 99% confidence.

From Exhibit 3, for the entire 1990 to 2014 sample period, estimates of Characteristic Exponents together with their 99% confidence interval ranges are 1.596 ±0.005 for all four property types combined, 1.629 ±0.011 for Office properties, 1.531 ±0.013 for Retail properties, 1.618 ±0.009 for Industrial properties, and 1.652 ±0.012 for Apartment properties.

Exhibit 6 displays the sample Characteristic Exponents α_t of both the aggregated and individual property type residuals. It appears that α_t could be time-invariant within property types. Furthermore, Exhibit 5 that shows graphical representations of these data suggests that α_t likely varies across property type.

Exhibit 5 shows the Characteristic Exponent for each property type and the aggregate over the full 1990 to 2014 time period along with the 99% confidence bands. In the case of the Characteristic Exponent, Office, Industrial, and Apartment property types are statistically indistinguishable from one another while Retail stands apart. These differences among property types deviate from conclusions of prior studies using non-MLE analytical techniques where more statistical similarities were observed.

The above analysis implies that over the sample period 1990 to 2014 (1) real estate investment risk was heteroscedastic for properties of a type and in the aggregate; (2) during virtually all sample subperiods and across property type, Levy-stable infinite-variance skewed asset-specific risk functions with a Characteristic Exponent α of approximately 1.596 with a 99% confidence interval range of ±0.005 modeled the observed distributions of return residuals better than Normally distributed risk candidates; and (3) property type differences in the Characteristic Exponent across property types are likely, which begs the question for further research into other dimensions along which distinct differences may emerge and investment strategies or tactics that might be employed to take advantage of these differences.

Conclusions

Once committed to investing in commercial real estate, return on investment comes in two forms: capital appreciation and periodic net cash flow. [7] MVI and FCFY are superior measures of these components rather than reformulations of the classic NPI that have been used for decades.

The MLE analysis of this study supports the conclusion that there is no single value for the Characteristic Exponent of asset-specific risk across property type. Nonetheless, all four NCREIF property types exhibit statistically significant departures from the Normal distribution at the 99% confidence level for all twenty-four years of this study.

As a practical matter, few, if any, investors have the financial resources to acquire commercial real estate portfolios even of a single property type to approach the risk reduction magnitudes available in other liquid, securitized asset classes like common stocks such that the non-systematic and systematic risks of the portfolio are comparable.

Once invested in real estate, the systematic risk is a fact of ownership. However, investors may affect for good or for bad the non-systematic, property-specific risk. Arguably, this is more so in the real estate asset class than in other investments where ownership and control are separated (albeit private equity is more akin to real estate in that its performance can be managed or directly impacted by the owner).[8]

Risk and reward are said to the positively related. But, risk, positive or negative, can be managed. Reward is just accounting for the degree to which risk has been achieved, again either positive or negative.

The analysis of this article implies that over the sample period 1990 to 2014 (1) real estate investment risk was heteroscedastic for properties of a type and in the aggregate; (2) during virtually all sample subperiods and across property type, Levy-stable infinite-variance skewed asset-specific risk functions with a Characteristic Exponent α of approximately 1.596 with a 99% confidence interval range of ± 0.005 modeled the observed distributions of return residuals better than Normally distributed risk candidates; and (3) property type differences in the Characteristic Exponent across property types are likely, which begs the question for further research into other dimensions along which distinct differences may emerge and investment strategies or tactics that might be employed to take advantage of these differences.

The conclusions of this study reinforce the earlier result that for institutional-grade real estate portfolios, the appropriate degree of risk reduction across multiple risk factors (locational, economic, etc.) could only be achieved by purchasing most of the institutional-grade properties in the U.S.—an obvious practical impossibility. This implies that institutional real estate portfolio management must be concerned with the asset-specific risk component of each property included in the portfolio with perhaps lesser consideration given to market/systematic and market-sector risk components. In street parlance, this means that your real estate eggs are always in one basket, so it behoves you to watch those individual eggs very carefully. This elevates in importance the otherwise pedestrian tasks of on-the-ground management and enterpreneurship.

In this study, property type differences in performance matter, more so than demonstrated in previous studies.

The fundamental reasons for differences in property-type performance boil down to the sources of net revenue and the market forces impacting the properties. In essence, lease structure and obsolescence or technological change matter most. These are subjects worthy of future research on specific property types irrespective of return differences that may or may not exist across property types. NCREIF Indicators: Capital Performance and Property Operations data sets offer ample statistics to pursue this research.

Slowly, knowledge of the non-Normal characteristics of real estate return distributions has spawned inquiry into processes and applications beyond the well worn mean-variance models. For example, Brown (2004) notes differences in skewness of returns between institutional-grade commercial property and direct, private investment property where investors can influence the outcome. Commercial property returns, subject to bond-like leases for a share of their total value estimates, often exhibit bond-like negative skewness. By contrast, Brown finds that direct property returns are more likely to be positively skewed leading to the conjecture that the value-added activities of owners is, on balance, a net positive for performance.

Brown and Young (2011) offer a middle-ground solution to measuring downside risk in real estate by means of so-called Coherent Risk Measures, an improvement both quantitatively and qualitatively over the more familiar Value At Risk (VaR) measures used in banking institutions.

Mathematica and tools built upon the platform collectively called Wolfram Demonstrations Project offer quick, easy, and graphically interesting ways to probe aspects of return distributions and examine their consequences. For example, Brown's "Forming the Efficient Frontier When Returns are Non-Normal" demonstration shows efficient frontiers generated Normal versus Levy-stable distribution under user-selectable variations of Levy-stable distribution parameters.[9]

Applications of Levy-stable distribution parameters to real estate return distributions to date have generally involved proprietary or difficult-to-acquire databases. NCREIF data from the new MVI and FCFY series are available to members pursuant to a "disaggregation request." Investment Property Databank (IPD) data required processing by an employee of IPD but the data in raw form were unavailable to the other researchers on the study. These restrictive procedures encumber would-be researchers and doubtless limit the investigations that might afford more and varied insights into the nature and implications of real estate return distributions. We can hope that the efforts of those who have been able to penetrate the labyrinth thus far will convince database gatekeepers that openess and transparency can benefit the entire industry.

Lastly, it may be time to re-think the entire enterprise of trying to specify the shape of the distribution in order to derive two or four parameters. The on-going revolution in Big Data, machine learning, click-throughs and apps have erased the stigma previously associated with data mining. Maximum entropy theory, which asserts that the probability distribution that most likely describes the current state of knowledge is the one with the largest entropy, is perhaps the most used optimization technique in data analysis today as discrete Bayesian approaches replace MLE and continuous frequentist methods.

If real estate exists at the intersection of law and economics, its data span alphabetic and numeric. This means that Natural Language Processing may soon play a role as big as spreadsheets do now. As skilled hackers in a foreign country can liberate data overnight in a variety of ways from any number of sources, one wonders when incarcerating data will go the way of locking Pilgrims in stocks. What we have today is science being turned on its head: data determines theory, not the other way around. Real estate has long suffered from a reputation of having hundreds of years of history unimpeded by progress. That needs to change and, we believe, research in the real estate asset class would benefit from progress in this direction.

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Exhibit 1
Comparison of the Number of Observations by Year
For the NCREIF Property Index (NPI) and new NCREIF Indicators (MVI+FCFY)

All Properties Combined:

Year or Period	NPI Number of Obs.	MVI-FCFY Number of Obs.	Difference (col. 1 – col. 2) Number	Percent
2010	5,455	5,291	164	3.0%
2009	5,588	5,507	81	1.4
2008	5,552	5,443	109	2.0
2007	4,702	4,643	59	1.3
2006	4,019	3,949	70	1.7
2005	3,383	3,325	58	1.7
2004	3,266	3,204	62	1.9
2003	3,236	3,171	65	2.0
2002	3,070	3,036	34	1.1
2001	2,636	2,610	26	1.0
2000	2,292	2,249	43	1.9
1999	2,059	2,019	40	1.9
1998	1,885	1,857	28	1.5
1997	1,859	1,841	18	1.0
1996	1,925	1,889	36	1.9
1995	1,727	1,680	47	2.7
1994	1,707	1,657	50	2.9
1993	1,848	1,808	40	2.2
1992	1,907	1,868	39	2.0
1991	1,825	1,788	37	2.0
1990	1,595	1,561	34	2.1
1990-2010	61,536	60,396	1,140	1.9%

By Property Types:

Office	1990-2010	15,841	15,560	281	1.8%
Retail	1990-2010	10,595	10,373	222	2.1%
Industrial	1990-2010	22,410	21,943	467	2.1%
Apartment	1990-2010	12,690	12,520	170	1.3%

Exhibit 2

Comparison of MLE Stable Distribution Characteristic Exponent α
for All Properties Combined and for Office Properties
computed on NCREIF NPI Dataset (columns 1 & 3)
and on NCREIF MVI+FCFY Dataset (columns 2 & 4)

	All Properties Combined			Office Properties		
Year	col. 1 α	col. 2 α	col. 2 – col. 1	col. 3 α	col. 4 α	col. 4 – col. 3
2010	1.644	1.688	0.044	1.587	1.651	0.064
2009	1.708	1.739	0.031	1.734	1.761	0.029
2008	1.691	1.723	0.032	1.689	1.696	0.007
2007	1.555	1.556	0.001	1.574	1.581	0.007
2006	1.556	1.565	0.009	1.542	1.544	0.002
2005	1.643	1.647	0.004	1.463	1.579	0.116
2004	1.668	1.668	0.000	1.610	1.596	–0.014
2003	1.574	1.584	0.010	1.463	1.467	0.004
2002	1.544	1.547	0.003	1.413	1.422	0.009
2001	1.416	1.423	0.007	1.477	1.471	–0.006
2000	1.398	1.399	0.001	1.285	1.297	0.012
1999	1.416	1.437	0.021	1.534	1.571	0.037
1998	1.531	1.527	–0.004	1.434	1.411	–0.023
1997	1.496	1.507	0.011	1.590	1.585	–0.005
1996	1.474	1.487	0.013	1.715	1.700	–0.015
1995	1.484	1.495	0.011	1.437	1.472	0.035
1994	1.433	1.467	0.034	1.475	1.475	0.000
1993	1.446	1.451	0.005	1.423	1.445	0.022
1992	1.500	1.523	0.023	1.478	1.528	0.050
1991	1.593	1.592	–0.001	1.440	2.000	0.560
1990	1.464	1.466	0.002	1.388	1.388	0.000
average	1.535	1.547	0.012	1.512	1.554	0.042

Exhibit 2 (continued)
Comparison of MLE Stable Distribution Characteristic Exponent α
for Retail, Industrial, and Apartment Properties
computed on NCREIF NPI Dataset (columns 1, 3 & 5)
and on NCREIF MVI+FCFY Dataset (columns 2, 4 & 6)

	Retail Properties			Industrial Properties			Apartment Properties		
	col. 1	col. 2	col. 2	col. 3	col. 4	col. 4	col. 5	col.6	col. 6
Year	α	α	– col. 1	α	α	– col. 3	α	α	– col. 5
2010	1.522	1.593	0.071	1.637	1.685	0.048	1.787	1.801	0.014
2009	1.627	1.678	0.051	1.661	1.675	0.014	1.798	1.848	0.050
2008	1.660	1.687	0.027	1.589	1.632	0.043	1.843	1.862	0.019
2007	1.432	1.427	–0.005	1.658	1.656	–0.001	1.477	1.477	0.000
2006	1.543	1.538	–0.005	1.637	1.652	0.015	1.553	1.572	0.019
2005	1.658	1.691	0.033	1.767	1.767	0.000	1.458	1.463	0.005
2004	1.649	1.648	–0.001	1.649	1.653	0.004	1.653	1.661	0.008
2003	1.565	1.586	0.021	1.514	1.524	0.010	1.650	1.661	0.011
2002	1.364	1.381	0.017	1.360	1.355	–0.005	1.755	1.757	0.002
2001	1.204	1.236	0.032	1.287	1.293	0.006	1.748	1.752	0.004
2000	1.227	1.237	0.010	1.252	1.269	0.017	1.670	1.670	0.000
1999	1.445	1.458	0.013	1.214	1.232	0.018	1.483	1.500	0.017
1998	1.412	1.395	–0.017	1.441	1.433	–0.008	1.642	1.649	0.007
1997	1.207	1.212	0.005	1.479	1.494	0.015	1.487	1.505	0.018
1996	1.064	1.081	0.017	1.360	1.376	0.016	1.555	1.559	0.004
1995	1.172	1.177	0.005	1.538	1.534	–0.004	1.474	1.554	0.070
1994	1.172	1.217	0.045	1.536	1.568	0.032	1.673	1.674	0.001
1993	1.198	1.191	–0.007	1.303	1.320	0.017	1.260	1.260	0.000
1992	1.341	1.324	–0.017	1.293	1.329	0.036	1.177	1.168	–0.009
1991	1.552	1.557	0.005	1.413	1.391	–0.022	1.372	1.379	0.007
1990	1.251	1.250	–0.001	1.202	1.216	0.014	1.793	1.794	0.001
average	1.394	1.408	0.014	1.466	1.479	0.013	1.586	1.598	0.012

Exhibit 3
MLE Levy-stable Distribution Parameters
for NCREIF MVI+FCFY Series by Property Type
Log Annual Total Return Residuals & Mean Returns & Number of Observations
1990 to 2014

Property Type	α	β	γ	Mean Return	Number of Observations
Office	1.629 **	-0.288	0.071	0.107	20,330
Retail	1.531 **	-0.162	0.052	0.098	14,274
Industrial	1.618 **	-0.257	0.061	0.098	31,380
Apartment	1.652 **	0.127	0.051	0.098	17,559
All Properties	1.596 **	-0.159	0.059	0.098	83,543

Statistically significant confidence of non-Normality $\alpha \neq 2.0$): ** = 99% confidence

α is the Characteristic Exponent, and only equals 2.0 for the Normal distribution
β is the Skewness Parameter in the range -1.0 to +1.0
γ is the (positive) Scale Parameter which measures the spread of the distribution about δ

Note: The mean returns are shown in Exhibit 3 simply for purposes of completeness.

Exhibit 4
Dispersion Estimates for NCREIF MVI+FCFY Series
Standard Deviation, Semi-Interquartile Range, and Scale Parameter γ
1990 to 2014

Property Type	Std Dev	Semi-IQR	γ	Semi-IQR minus γ	Std Dev minus γ
Office	0.135	0.065	0.071	-0.006	0.064
Retail	0.112	0.048	0.052	-0.004	0.060
Industrial	0.118	0.056	0.061	-0.005	0.057
Apartment	0.093	0.047	0.051	-0.004	0.042
All Properties	0.117	0.055	0.059	-0.004	0.058

Exhibit 5
MLE Stable Distribution Parameters for NCREIF MVI+FCFY Series by Property Type
1990 to 2014
(bands indicate 99% confidence interval)

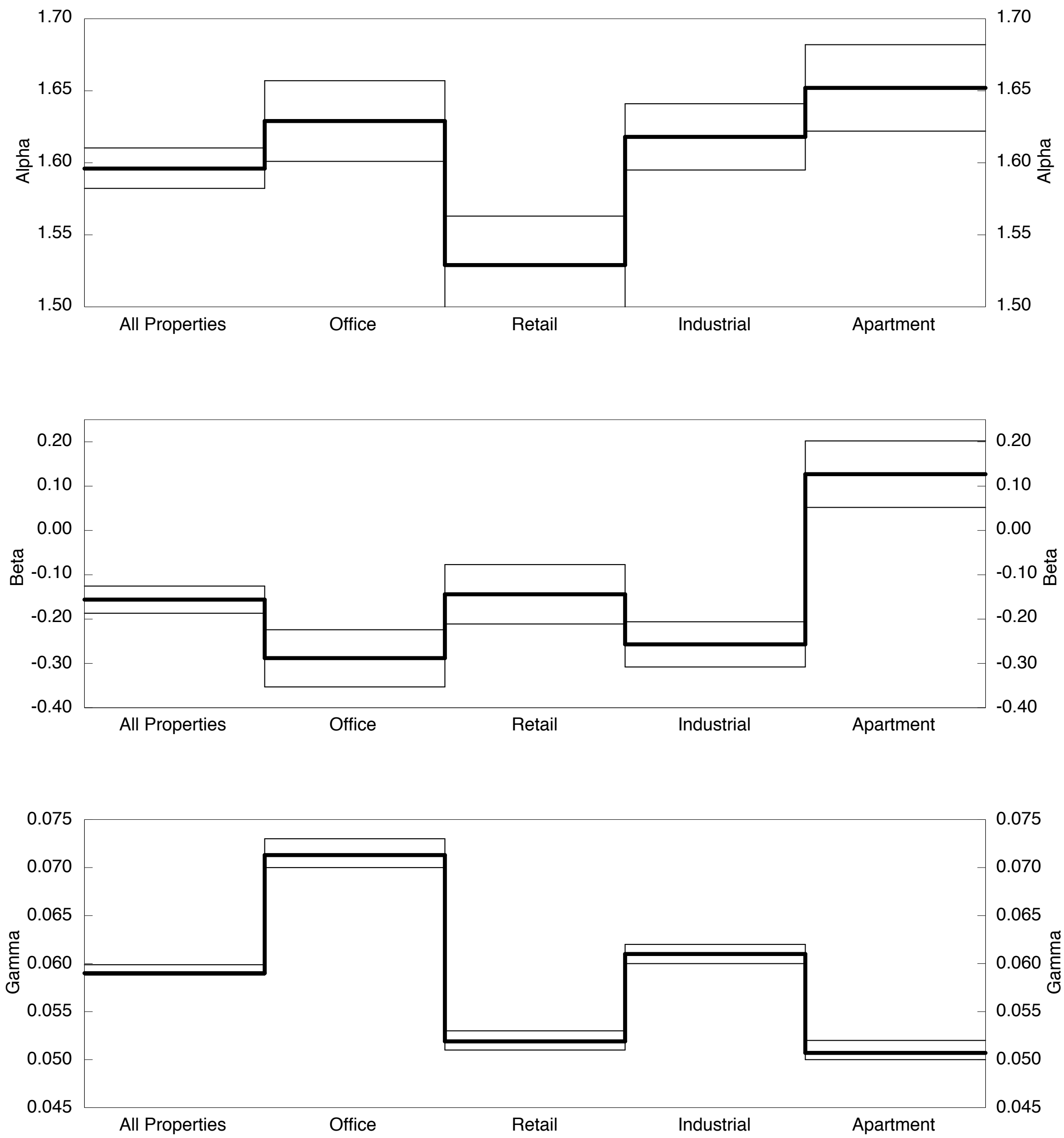


Exhibit 6
MLE Levy-stable Distribution Parameters for NCREIF MVI+FCFY Series
Log Annual Total Return Residuals & Mean Returns & Number of Observations

All Properties Combined:

Year or Period	α	β	γ	Mean Return	Number of Obs.
2014	1.526 **	0.123	0.044	0.107	5,866
2013	1.522 **	0.000	0.045	0.098	5,923
2012	1.529 **	-0.118	0.050	0.090	5,994
2011	1.666 **	-0.179	0.062	0.112	5,364
2010	1.688 **	-0.478	0.073	0.092	5,291
2009	1.739 **	-0.797	0.094	-0.182	5,507
2008	1.723 **	-0.655	0.079	-0.065	5,443
2007	1.556 **	0.374	0.047	0.117	4,643
2006	1.565 **	0.379	0.054	0.137	3,949
2005	1.647 **	0.364	0.064	0.157	3,325
2004	1.668 **	-0.239	0.057	0.111	3,204
2003	1.584 **	-0.531	0.052	0.072	3,171
2002	1.547 **	-0.550	0.052	0.058	3,036
2001	1.423 **	-0.350	0.041	0.072	2,610
2000	1.399 **	0.255	0.039	0.112	2,249
1999	1.437 **	0.171	0.037	0.107	2,019
1998	1.527 **	0.522	0.047	0.139	1,857
1997	1.507 **	0.402	0.050	0.134	1,841
1996	1.487 **	0.084	0.047	0.106	1,889
1995	1.495 **	-0.226	0.051	0.093	1,680
1994	1.467 **	-0.322	0.055	0.075	1,657
1993	1.451 **	-0.661	0.067	0.018	1,808
1992	1.523 #	-0.912	0.077	-0.037	1,868
1991	1.592 #	-0.964	0.083	-0.058	1,788
1990	1.466 **	-0.833	0.061	-0.001	1,561
1990-14	1.596 **	-0.159	0.059	0.064	83,543
99% conf.	0.005				

Exhibit 6 (continued)
MLE Levy-stable Distribution Parameters for NCREIF MVI+FCFY Series
Log Annual Total Return Residuals & Mean Returns & Number of Observations

Office Properties:

Year or Period	α	β	γ	Mean Return	Number of Obs.
2014	1.547 **	-0.235	0.053	0.086	1,156
2013	1.479 **	-0.163	0.048	0.078	1,185
2012	1.517 **	-0.161	0.050	0.070	1,209
2011	1.632 **	-0.142	0.066	0.094	1,220
2010	1.651 **	-0.375	0.078	0.066	1,249
2009	1.761 **	-0.813	0.111	-0.213	1,370
2008	1.696 **	-0.724	0.086	-0.076	1,363
2007	1.581 **	0.414	0.063	0.138	1,077
2006	1.544 **	0.319	0.064	0.140	958
2005	1.579 **	0.214	0.066	0.148	897
2004	1.596 **	-0.454	0.063	0.083	913
2003	1.467 **	-0.762	0.058	0.033	950
2002	1.422 **	-0.802	0.060	0.012	921
2001	1.471 **	-0.552	0.052	0.048	822
2000	1.297 **	0.349	0.042	0.118	648
1999	1.571 **	0.374	0.043	0.112	576
1998	1.411 **	0.814	0.054	0.171	490
1997	1.585 **	0.611	0.069	0.182	402
1996	1.700 **	0.287	0.064	0.127	424
1995	1.472 **	-0.233	0.068	0.078	369
1994	1.475 **	-0.475	0.078	0.051	400
1993	1.445 **	-0.833	0.083	-0.031	451
1992	1.528 #	-0.966	0.098	-0.107	435
1991	2.000 #	0.626	0.138	-0.146	443
1990	1.388 #	-0.955	0.075	-0.070	402
1990-14	1.629 **	-0.288	0.071	0.048	20,330
99% conf.	0.011				

Exhibit 6 (continued)
MLE Levy-stable Distribution Parameters for NCREIF MVI+FCFY Series
Log Annual Total Return Residuals & Mean Returns & Number of Observations

Retail Properties:

Year or Period	α	β	γ	Mean Return	Number of Obs.
2014	1.463 **	0.248	0.040	0.112	995
2013	1.350 **	0.214	0.041	0.106	944
2012	1.414 **	0.000	0.046	0.094	1,005
2011	1.717 **	-0.125	0.065	0.112	957
2010	1.593 **	-0.537	0.066	0.084	881
2009	1.678 **	-0.776	0.080	-0.144	813
2008	1.687 **	-0.724	0.076	-0.064	864
2007	1.427 **	0.497	0.037	0.107	710
2006	1.538 **	0.646	0.037	0.119	560
2005	1.691 **	0.604	0.056	0.169	447
2004	1.648 **	0.351	0.052	0.172	454
2003	1.586 **	-0.096	0.042	0.132	418
2002	1.381 **	0.095	0.036	0.107	446
2001	1.236 **	-0.395	0.030	0.078	446
2000	1.237 **	-0.164	0.032	0.093	438
1999	1.458 **	0.000	0.035	0.104	405
1998	1.395 **	0.085	0.038	0.116	398
1997	1.212 **	-0.141	0.038	0.094	438
1996	1.081 **	-0.305	0.037	0.064	496
1995	1.177 **	-0.622	0.042	0.044	372
1994	1.217 **	-0.335	0.037	0.060	355
1993	1.191 **	-0.503	0.045	0.033	408
1992	1.324 #	-0.933	0.055	-0.006	377
1991	1.557 **	-0.818	0.065	-0.019	374
1990	1.250 **	-0.586	0.035	0.047	273
1990-14	1.531 **	-0.162	0.052	0.070	14,274
99% conf.	0.013				

Exhibit 6 (continued)
MLE Levy-stable Distribution Parameters for NCREIF MVI+FCFY Series
Log Annual Total Return Residuals & Mean Returns & Number of Observations

Industrial Properties:

Year or Period	α	β	γ	Mean Return	Number of Obs.
2014	1.537 **	0.137	0.047	0.120	2,469
2013	1.568 **	-0.094	0.051	0.104	2,550
2012	1.562 **	-0.244	0.058	0.089	2,500
2011	1.671 **	-0.241	0.066	0.104	1,918
2010	1.685 **	-0.597	0.074	0.072	1,929
2009	1.675 **	-0.570	0.094	-0.189	2,032
2008	1.632 **	-0.724	0.071	-0.060	1,910
2007	1.656 **	0.246	0.045	0.119	1,800
2006	1.652 **	0.301	0.057	0.143	1,660
2005	1.767 **	-0.079	0.069	0.161	1,317
2004	1.653 **	-0.528	0.060	0.108	1,167
2003	1.524 **	-0.519	0.052	0.076	1,126
2002	1.355 **	-0.600	0.044	0.064	984
2001	1.293 **	-0.223	0.034	0.085	625
2000	1.269 **	0.377	0.033	0.124	583
1999	1.232 **	0.068	0.043	0.108	564
1998	1.433 **	0.628	0.043	0.145	490
1997	1.494 **	0.678	0.050	0.147	610
1996	1.376 **	0.390	0.039	0.125	623
1995	1.534 **	0.000	0.045	0.118	639
1994	1.568 **	-0.290	0.053	0.080	613
1993	1.320 **	-0.773	0.060	0.010	681
1992	1.329 #	-0.912	0.070	-0.031	833
1991	1.391 #	-0.919	0.067	-0.036	774
1990	1.216 **	-0.770	0.050	0.006	723
1990-14	1.618 **	-0.257	0.061	0.067	31,380
99% conf.	0.009				

Exhibit 6 (continued)
MLE Levy-stable Distribution Parameters for NCREIF MVI+FCFY Series
Log Annual Total Return Residuals & Mean Returns & Number of Observations

Apartment Properties:

Year or Period	α	β	γ	Mean Return	Number of Obs.
2014	1.704 **	0.414	0.036	0.096	1,246
2013	1.786 **	0.380	0.035	0.099	1,244
2012	1.647 **	0.170	0.041	0.106	1,280
2011	1.753 **	-0.350	0.051	0.0141	1,269
2010	1.801 **	-0.254	0.071	0.157	1,232
2009	1.848 **	-0.806	0.083	-0.160	1,292
2008	1.862 **	-0.721	0.082	-0.061	1,306
2007	1.477 **	0.486	0.042	0.101	1,056
2006	1.572 **	0.524	0.051	0.135	771
2005	1.463 **	0.850	0.055	0.154	664
2004	1.661 **	0.684	0.044	0.110	670
2003	1.661 **	0.000	0.042	0.081	677
2002	1.757 **	0.148	0.047	0.077	685
2001	1.752 **	-0.350	0.044	0.082	592
2000	1.670 **	0.246	0.040	0.108	538
1999	1.500 **	0.366	0.034	0.103	455
1998	1.649 **	0.592	0.040	0.116	405
1997	1.505 **	0.468	0.036	0.109	391
1996	1.559 **	0.447	0.039	0.108	346
1995	1.554 **	0.744	0.037	0.119	300
1994	1.674 **	0.127	0.053	0.114	289
1993	1.260 **	0.221	0.046	0.099	268
1992	1.168 **	-0.736	0.038	0.028	223
1991	1.379 **	-0.845	0.059	-0.017	197
1990	1.794 #	-1.000	0.052	0.062	163
1990-14	1.652 **	0.127	0.051	0.077	17,559
99% conf.	0.012				

Statistically significant confidence of non-Normality α ≠ 2.0):
** = 99% confidence
# = indeterminate

α is the Characteristic Exponent, and only equals 2.0 for the Normal distribution
β is the Skewness Parameter in the range -1.0 to +1.0
γ is the (positive) Scale Parameter which measures the spread of the distribution about δ

Note: The mean returns are shown in Exhibit 6 simply for purposes of completeness.

Exhibit 7
Characteristic Exponent "Alpha" of MLE Stable Distributions for NCREIF MVI+FCFY Series
by Property Type, 1990 to 2014
(bands indicate 99% confidence interval)

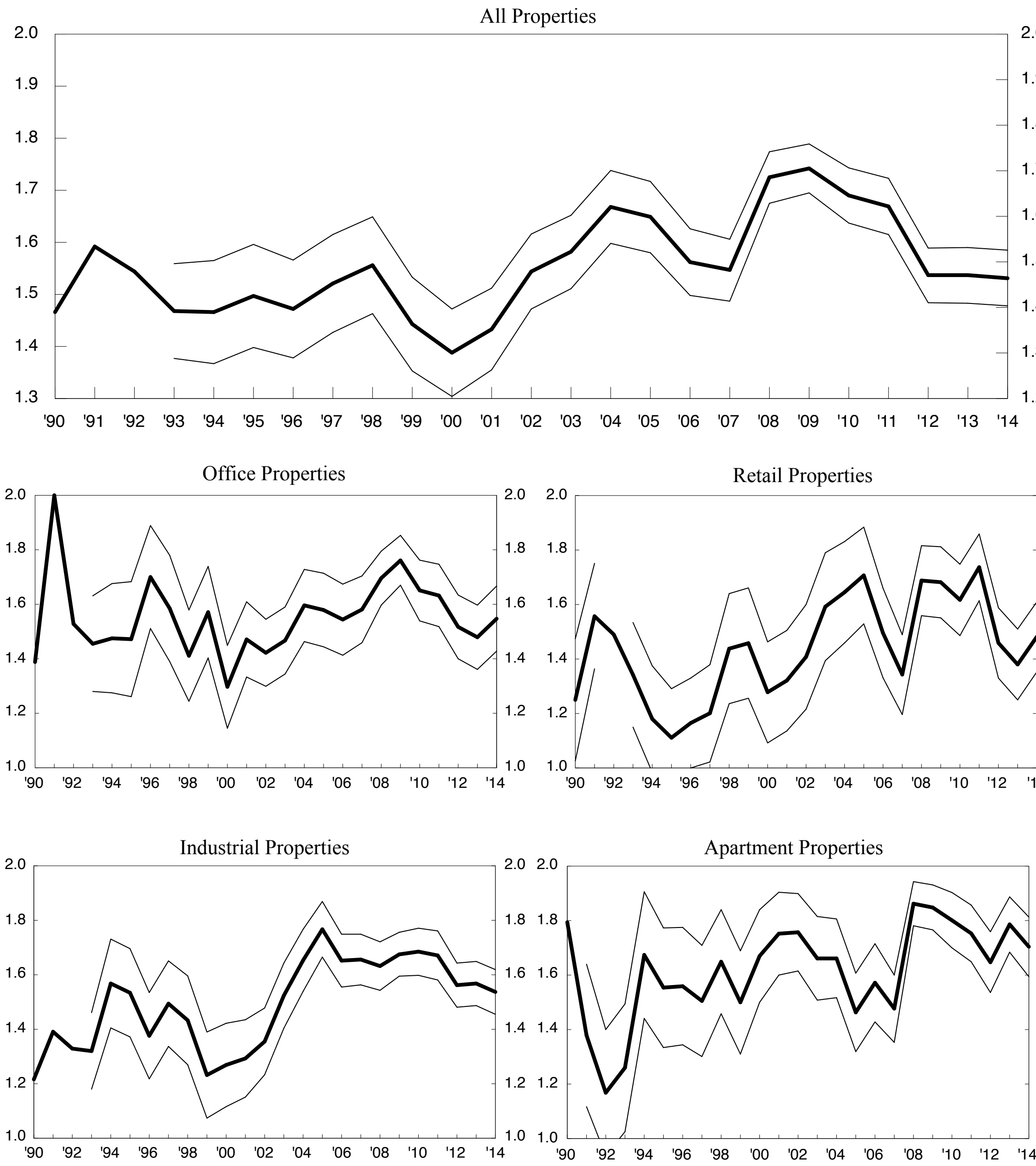


Exhibit 8
Skewness Parameter "Beta" of MLE Stable Distributions for NCREIF MVI+FCFY Series
by Property Type, 1990 to 2014
(bands indicate 99% confidence interval)

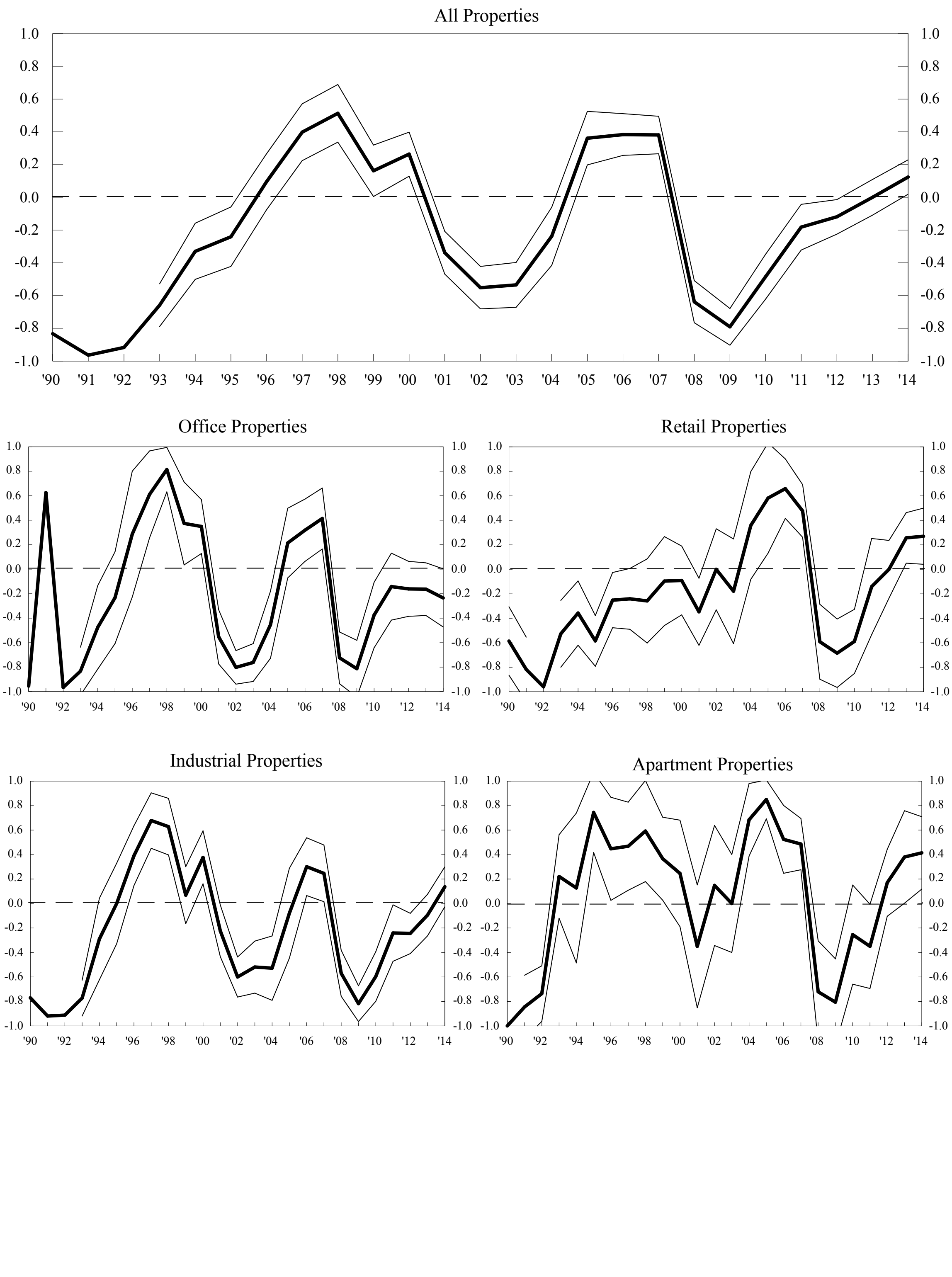


Exhibit 9
Scale Parameter "Gamma" of MLE Stable Distributions for NCREIF MVI+FCFY Series,
by Property Type, 1990 to 2014
(bands indicate 99% confidence interval)

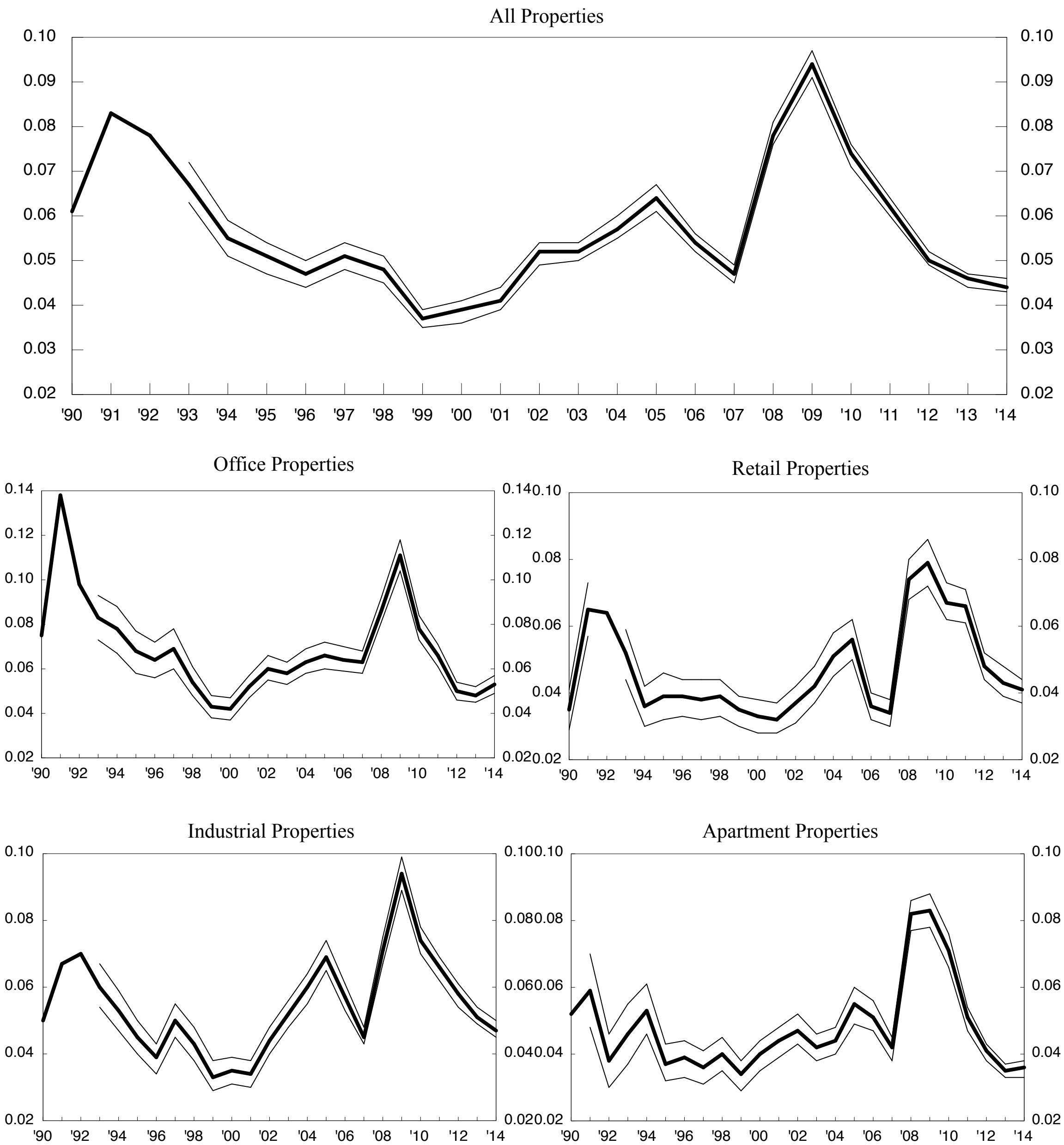


Exhibit 10a
Distribution of Log Annual Total Return Residuals
NCREIF MVI+FCFY Series, All Properties, 1990 to 2014

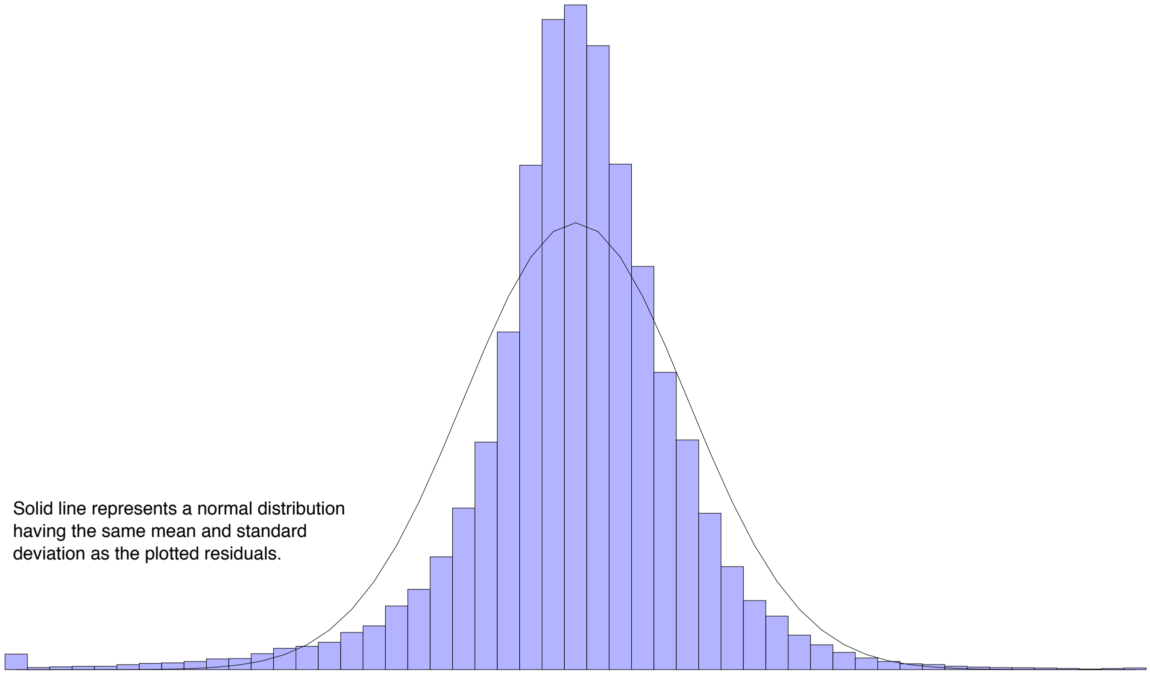


Exhibit 10b
Difference in Frequency, Log Annual Total Return Residuals to Normal Distribution
NCREIF MVI+FCFY Series, All Properties, 1990 to 2014

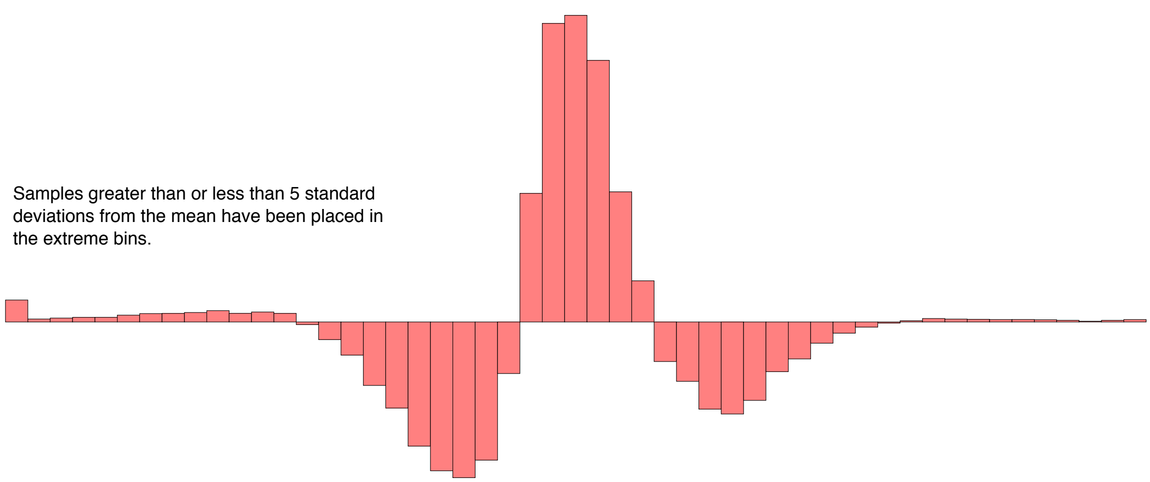


Exhibit 11
Dispersion Estimates for NCREIF MVI+FCFY Series
Standard Deviation, Semi-Interquartile Range, and Gamma

Office Properties:

Year or Period	Std Dev	Semi-IQ	γ	SemiIQ $-\gamma$	Number of Obs.
2014	0.114	0.046	0.053	-0.007	1,156
2013	0.111	0.045	0.048	-0.003	1,185
2012	0.114	0.047	0.050	-0.003	1,209
2011	0.122	0.059	0.066	-0.007	1,220
2010	0.145	0.067	0.078	-0.011	1,249
2009	0.181	0.109	0.111	-0.002	1,370
2008	0.150	0.088	0.086	0.002	1,363
2007	0.120	0.063	0.063	0.000	1,077
2006	0.130	0.063	0.064	-0.001	958
2005	0.132	0.066	0.066	0.000	897
2004	0.120	0.058	0.063	-0.005	913
2003	0.129	0.059	0.058	0.001	950
2002	0.131	0.066	0.060	0.006	921
2001	0.105	0.052	0.052	0.000	822
2000	0.104	0.041	0.042	-0.001	648
1999	0.084	0.041	0.043	-0.002	576
1998	0.118	0.064	0.054	0.010	490
1997	0.127	0.072	0.069	0.003	402
1996	0.113	0.061	0.064	-0.003	424
1995	0.141	0.061	0.068	-0.007	369
1994	0.153	0.072	0.078	-0.006	400
1993	0.174	0.090	0.083	0.007	451
1992	0.194	0.107	0.098	0.009	435
1991	0.196	0.131	0.138	-0.007	443
1990	0.171	0.088	0.075	0.012	402
1990-14	0.135	0.065	0.071	-0.006	20,330

Exhibit 11 (continued)
Dispersion Estimates for NCREIF MVI+FCFY Series
Standard Deviation, Semi-Interquartile Range, and Gamma

Retail Properties:

Year or Period	Std Dev	Semi-IQ	γ	SemiIQ $-\gamma$	Number of Obs.
2014	0.088	0.038	0.040	-0.002	995
2013	0.102	0.040	0.041	-0.001	944
2012	0.114	0.044	0.046	-0.002	1,005
2011	0.117	0.059	0.065	-0.006	957
2010	0.137	0.067	0.066	0.001	881
2009	0.143	0.067	0.080	-0.013	813
2008	0.134	0.069	0.076	-0.007	864
2007	0.084	0.038	0.037	0.001	710
2006	0.082	0.040	0.037	0.003	560
2005	0.098	0.056	0.056	0.000	447
2004	0.097	0.053	0.052	0.001	454
2003	0.104	0.040	0.042	-0.002	418
2002	0.084	0.033	0.036	-0.003	446
2001	0.096	0.031	0.030	0.001	446
2000	0.102	0.031	0.032	-0.001	438
1999	0.076	0.032	0.035	-0.003	405
1998	0.094	0.036	0.038	-0.002	398
1997	0.115	0.035	0.038	-0.003	438
1996	0.129	0.040	0.037	0.003	496
1995	0.126	0.045	0.042	0.003	372
1994	0.113	0.037	0.037	0.000	355
1993	0.146	0.053	0.045	0.008	408
1992	0.132	0.077	0.055	0.022	377
1991	0.126	0.070	0.065	0.005	374
1990	0.105	0.037	0.035	0.002	273
1990-14	0.112	0.048	0.052	-0.004	14,274

Exhibit 11 (continued)
Dispersion Estimates for NCREIF MVI+FCFY Series
Standard Deviation, Semi-Interquartile Range, and Gamma

Industrial Properties:

Year or Period	Std Dev	Semi-IQ	γ	SemiIQ $- \gamma$	Number of Obs.
2014	0.078	0.040	0.047	-0.007	2,469
2013	0.071	0.039	0.051	-0.012	2,550
2012	0.077	0.042	0.058	-0.016	2,500
2011	0.129	0.060	0.066	-0.006	1,918
2010	0.133	0.068	0.074	-0.006	1,929
2009	0.140	0.091	0.094	-0.003	2,032
2008	0.127	0.069	0.071	-0.002	1,910
2007	0.087	0.042	0.045	-0.003	1,800
2006	0.105	0.054	0.057	-0.003	1,660
2005	0.115	0.063	0.069	-0.006	1,317
2004	0.113	0.053	0.060	-0.007	1,167
2003	0.106	0.048	0.052	-0.004	1,126
2002	0.106	0.044	0.044	0.000	984
2001	0.096	0.034	0.034	0.000	625
2000	0.097	0.034	0.033	0.001	583
1999	0.096	0.030	0.043	-0.013	564
1998	0.093	0.044	0.043	0.001	490
1997	0.104	0.052	0.050	0.002	610
1996	0.092	0.038	0.039	-0.001	623
1995	0.090	0.041	0.045	-0.004	639
1994	0.105	0.048	0.053	-0.005	613
1993	0.141	0.075	0.060	0.015	681
1992	0.155	0.091	0.070	0.021	833
1991	0.147	0.084	0.067	0.017	774
1990	0.136	0.065	0.050	0.015	723
1990-14	0.118	0.056	0.061	-0.005	31,380

Exhibit 11 (continued)
Dispersion Estimates for NCREIF MVI+FCFY Series
Standard Deviation, Semi-Interquartile Range, and Gamma

Apartment Properties:

Year or Period	Std Dev	Semi-IQ	γ	SemiIQ $-\gamma$	Number of Obs.
2014	0.065	0.035	0.036	-0.001	1,246
2013	0.059	0.034	0.035	-0.001	1,244
2012	0.081	0.040	0.041	-0.001	1,280
2011	0.085	0.049	0.051	-0.002	1,269
2010	0.121	0.068	0.071	-0.003	1,232
2009	0.130	0.079	0.083	-0.004	1,292
2008	0.125	0.081	0.082	-0.001	1,306
2007	0.084	0.043	0.042	0.001	1,056
2006	0.097	0.054	0.051	0.003	771
2005	0.116	0.060	0.055	0.005	664
2004	0.078	0.046	0.044	0.002	670
2003	0.075	0.038	0.042	-0.004	677
2002	0.078	0.040	0.047	-0.007	685
2001	0.084	0.038	0.044	-0.006	592
2000	0.071	0.034	0.040	-0.006	538
1999	0.066	0.032	0.034	-0.002	455
1998	0.071	0.036	0.040	-0.004	405
1997	0.078	0.031	0.036	-0.005	391
1996	0.074	0.034	0.039	-0.005	346
1995	0.080	0.038	0.037	0.001	300
1994	0.096	0.046	0.053	-0.007	289
1993	0.118	0.044	0.046	-0.002	268
1992	0.089	0.055	0.038	0.017	223
1991	0.130	0.071	0.059	0.012	197
1990	0.082	0.040	0.052	-0.012	163
1990-14	0.093	0.047	0.051	-0.004	17,559

Exhibit 12a
Risk Reduction for Various α and Number of Assets
Curves Represent α from 2.00 to 1.50 in Increments of 0.10

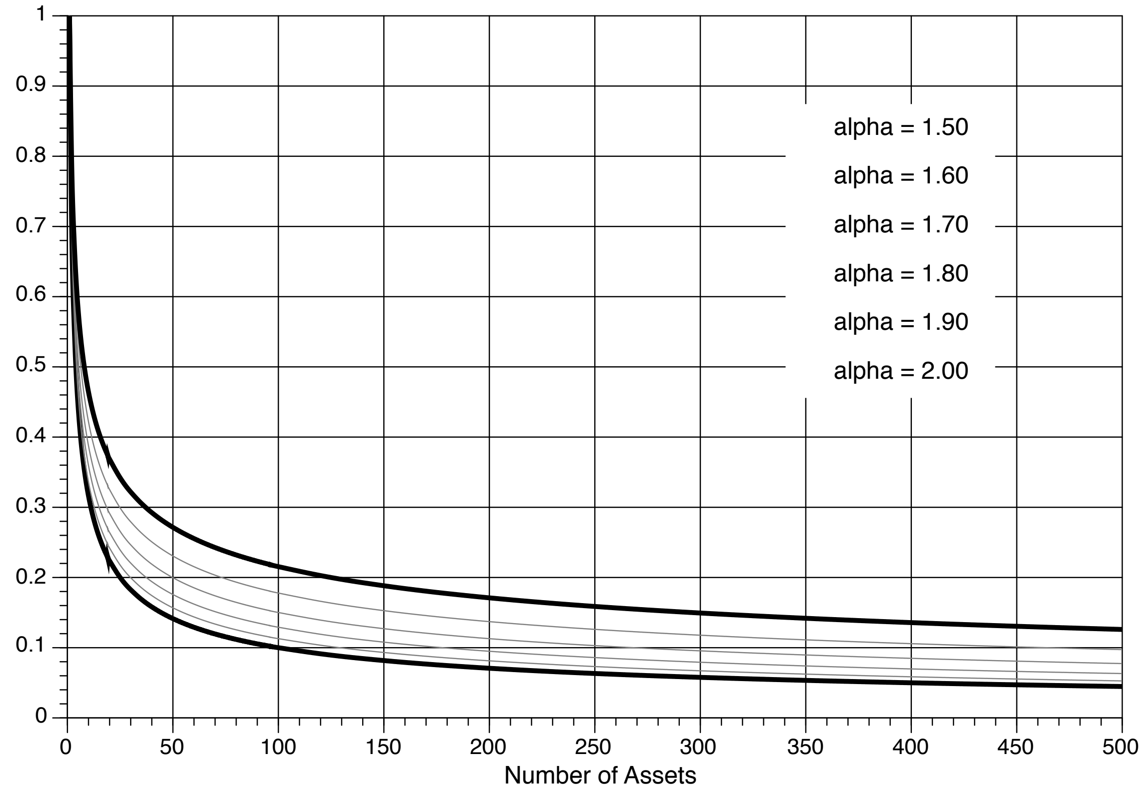
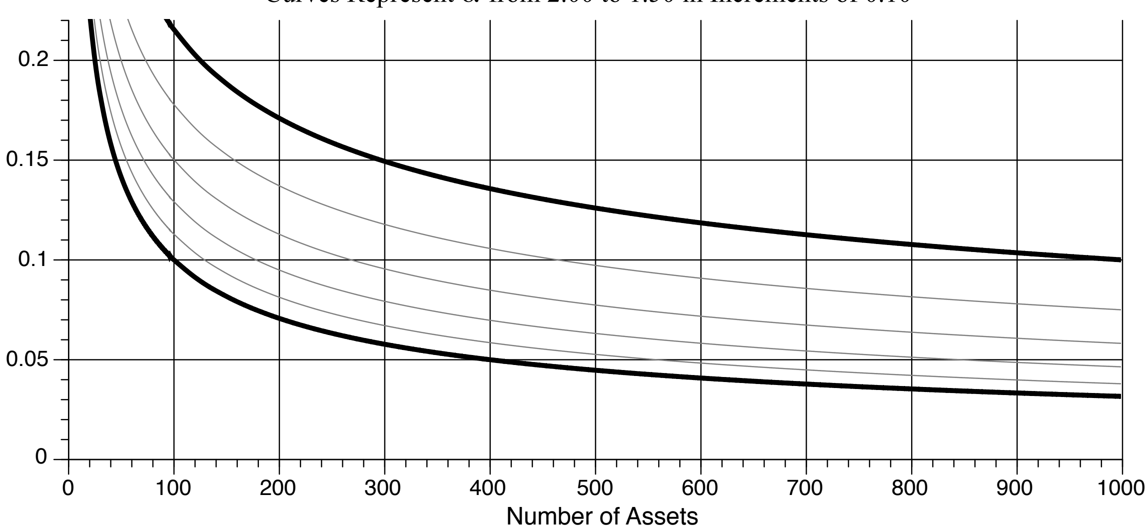


Exhibit 12b
Risk Reduction for Various α and Number of Assets
Curves Represent α from 2.00 to 1.50 in Increments of 0.10



1 The implementation of other analytical techniques up until the availability of Maximum Likelihood Estimators (MLE) for Levy-stable distributions is related in Young, and Graff (1995).

2 Less frequently there are also reporting problems such as recording a downpayment as the initial market value or as the sales price, the ending market value. These cause extreme distortion of quarterly returns for individual properties, but are largely obscured in the aggregate NPI returns commonly cited as representative of the asset class. However, when working with individual property returns or smaller aggregations of property returns as in this study, these problems would necessarily distort the return distribution statistics as they unfortunately did in earlier NPI-based studies.

3 Perhaps it should be noted that there have been other attempts to test the null hypothesis that real estate return distributions are Gaussian Normal using more conventional statistical techniques. The authors know of no cases that resulted in failing to reject the null. For example, there have been studies in the U.S. and even more in the U.K. using Chi-Square, Kolmogorov-Smirnov, or Anderson-Darling tests of common distributions like Logistic, Normal, Student's t, or Extreme Value. For a summary of these studies pre-2000, see Lizieri and Ward (2001).

4 It may be worth noting that the numerators of the Price and Cash Flow formulas are those originally proposed by Young, Geltner, McIntosh, and Poutasse (1995 and 1996) as replacements for the so-called Capital and Income Returns. Since NCREIF did not adopt the changes and retained the original formulation of Capital and Income Returns, the new Price and Cash Flow formulas were introduced for researchers interested in the Young, Geltner, McIntosh, and Poutasse concept. Notice too that the authors also proposed changing the NPI Total Return, Income Return, and Capital Return denominator to simply the beginning quarter's market value.

5 Examples of Partial Sales (PS) include the net sales price of one building from say a multi-building industrial park or the net sales price of an outparcel on the periphery of a shopping center.

6 Capital expenditures are generally reported as positive numbers, but occasionally there will be accounting "reversals" resulting in negative numbers for reported capital expenditures in a particular period. Some reversals may result from journal entries that reclassify or move outlays between periods.

7 Each of these have different risk characteristics per Brown (1998)

8 Passive investment in equity real estate is a fool's errand. Those who think they can invest passively in real estate by buying shares of REITs soon learn they have just bought stock.

9 The astute observer will immediately recognize a paradox in that efficient frontier graphics constitute a parametric plot that requires a covariance matrix. If Levy-stable distributions have no variance, they can have no covariances. One must remember, however, that Levy-stable distributions lack a variance in the limit. All finite samples have a variance that can be calculated. The demonstration illustrates the shape of the "frontier" using samples that are presumed to be drawn from a Levy-stable population having parameters supplied by the user. The demonstration is located at:

http://demonstrations.wolfram.com/FormingTheEfficientFrontierWhenReturnsAreNonNormal/.